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ON THE RATE OF GROWTH OF THE POPULATION OF THE UNITED STATES SINCE 1790 AND ITS MATHEMATICAL REPRESENTATION[1]

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It is obviously possible in any country or community of reasonable size to determine an empirical equation, by ordinary methods of curve fitting, which will describe the normal rate of population growth. Such a determination will not necessarily give any inkling whatever as to the underlying organic laws of population growth in a particular community. It will simply give a rather exact empirical statement of the nature of the changes which have occurred in the past. No process of empirically graduating raw data with a curve can in and of itself demonstrate the fundamental law which causes the occurring change.[2] In spite of the fact that such mathematical expressions of population growth are purely empirical, they have a distinct and considerable usefulness. This usefulness arises out of the fact that actual counts of population by census methods are made at only relatively infrequent intervals, usually 10 years and practically never oftener than 5 years. For many statistical purposes, it is necessary to have as accurate an estimate as possible of the population in inter-censal years. This applies not only to the years following that on which the last census was taken, but also to the inter-censal years lying between prior censuses. For purposes of practical statistics it is highly important to have these inter-censal estimates of population as accurate as possible, particularly for the use of the vital statistician, who must have these figures for the calculation of annual death rates, birth rates, and the like.

The usual method followed by census offices in determining the population in inter-censal years is of one or the other of two sorts, namely, by arithmetic progression or geometric progression. These methods assume that for any given short period of time the population is increasing either in arithmetic or geometric ratio. Neither of these assumptions is ever absolutely accurate even for short intervals of time, and both are grossly

inaccurate for the United States, at least, for any considerable period of time. What actually happens is that following any census estimates are made by one or another of these methods of the population for each year up to the next census, on the basis of data given by the last two censuses only. When that next census has been made, the previous estimates of the inter-censal years are corrected and adjusted on the basis of the facts brought out at that census period.

Obviously the best general method of estimating population in inter-censal years is that of fitting an appropriate curve to *all* the available data, and extrapolating for years beyond the last census, and reading off from the curve values for inter-censal years falling between earlier censuses. The methods of arithmetic or geometric progression use only two census counts at the most. Fitting a curve to all the known data regarding population by the method of least squares must obviously give a much sounder and more accurate result. In making this statement, one realizes perfectly, of course, the dangers of extrapolation. These dangers have been well emphasized by Perrin,[3] who used higher order parabolas to predict the future population of Buenos Aires. In keeping sharply before our minds the dangers of extrapolation from a curve, we are apt to forget that the methods of extrapolation by arithmetic or geometric progression have much less general validity than from a curve, and the inaccuracies are found in practice, except by the rarest of accidents, to be actually greater.

The first one to attempt an adequate mathematical representation of the normal rate of growth of the population of the United States was Pritchett.[4] Taking the census data from 1790 to 1880, inclusive, Pritchett fitted by the method of least squares the following equation:

$$P = A + Bt + Ct^2 + Dt^3 \qquad \text{(i)}$$

where P represents the population and t the time from some assumed epoch. As a matter of fact, Pritchett took the origin of the curve at 1840, practically the center of the series. With this third-order parabola Pritchett got a very accurate representation of the population between the dates covered. As will presently appear this curve did not give, even within the period covered, as accurate results as a more adequate curve would have done, and it overestimated the population after a very short interval beyond the last observed ordinate as is shown in table 2.

Some 13 years ago one of the writers[5] demonstrated the applicability of a logarithmic curve of the form

$$y = a + bx + cx^2 + d \log x \qquad \text{(ii)}$$

to the representation of growth changes, using the aquatic plant *Ceratophyllum* as material. Following the application of this curve to growth of this plant it was found equally useful in representing a wide range of other growth and related changes.[6] This list now includes, of matters

worked out in the Biological Laboratory of the Maine Experiment Station, such diverse phenomena as change of size of egg with successive layings, change of milk production with age, etc. Donaldson and Hatai[7] have demonstrated the applicability of this type of equation to bodily growth in the white rat and frog.

While the increase in size of a population cannot on *a priori* grounds be regarded, except by rather loose analogy, as the same thing as the growth of an organism in size, nevertheless it is essentially a growth phenomenon. It, therefore, seems entirely reasonable that this type of curve should give a more adequate representation of population increase than a simple third-order parabola. The actual event justifies this assumption, as will presently appear.

Table 1 shows the counted population as determined by the Census Bureau on the dates mentioned from 1790 to 1910. The exact dates were furnished in a personal communication from the present Director of the Census. These figures embody some adjustments and corrections made by the Census Bureau since the original censuses were made.

TABLE 1

SHOWING THE DATES OF THE TAKING OF THE CENSUS AND THE RECORDED POPULATIONS FROM 1790 TO 1910

DATE OF CENSUS		RECORDED POPULATION (REVISED FIGURES FROM STATISTICAL ABST., 1918)
Year	Month and Day	
1790	First Monday in August	3,929,214
1800	First Monday in August	5,308,483
1810	First Monday in August	7,239,881
1820	First Monday in August	9,638,453
1830	June 1	12,866,020
1840	June 1	17,069,453
1850	June 1	23,191,876
1860	June 1	31,443,321
1870	June 1	38,558,371
1880	June 1	50,155,783
1890	June 1	62,947,714
1900	June 1	75,994,575
1910	April 15	91,972,266

To the data of table 1 the following equation was fitted by the method of least squares, taking origin at 1780, and making due allowance in the abscissal intervals for the actual dates of the several censuses:

$$y = a + bx + cx^2 + d \log x$$

where y denotes population and x time. The actual equation deduced was

$$y = 9{,}064{,}900 - 6{,}281{,}430x + 842{,}377x^2 + 19{,}829{,}500 \log x. \quad \text{(iii)}$$

The results are set forth in table 2, where Pritchett's figures are given for comparison.

TABLE 2

SHOWING (*a*) THE ACTUAL POPULATION[1] ON CENSUS DATES, (*b*) ESTIMATED POPULATION FROM PRITCHETT'S THIRD-ORDER PARABOLA, (*c*) ESTIMATED POPULATION FROM LOGARITHMIC PARABOLA, AND (*d*) (*e*) ROOT-MEAN SQUARE ERRORS OF BOTH METHODS

CENSUS YEAR	(*a*) OBSERVED POPULATION	(*b*) PRITCHETT ESTIMATE	(*c*) LOGARITHMIC PARABOLA ESTIMATE	(*d*) ERROR OF (*b*)	(*e*) ERROR OF (*c*)
1790	3,929,000	4,012,000	3,693,000	+ 83,000	− 236,000
1800	5,308,000	5,267,000	5,865,000	− 41,000	+ 557,000
1810	7,240,000	7,059,000	7,293,000	− 181,000	+ 53,000
1820	9,638,000	9,571,000	9,404,000	− 67,000	− 234,000
1830	12,866,000	12,985,000	12,577,000	+ 119,000	− 289,000
1840	17,069,000	17,484,000	17,132,000	+ 415,000	+ 63,000
1850	23,192,000	23,250,000	23,129,000	+ 58,000	− 63,000
1860	31,443,000	30,465,000	30,633,000	− 978,000	− 810,000
1870	38,558,000	39,313,000	39,687,000	+ 755,000	+1,129,000
1880	50,156,000	49,975,000	50,318,000	− 181,000	+ 162,000
1890	62,948,000	62,634,000	62,547,000	− 314,000	− 401,000
1900	75,995,000	77,472,000	76,389,000	+1,477,000	+ 394,000
1910	91,972,000	94,673,000	91,647,000	+2,701,000	− 325,000
				935,000[2]	472,000[2]
1920		114,416,000	108,214,000		

[1] To the nearest thousand.

[2] Root-mean square error.

It is obvious from the data of table 2 that, with the same number of constants, the logarithmic parabola gives a distinctly better graduation than a third-order parabola.

The extreme precision of the present graduation is shown graphically in figure 1.

It is evident that as a purely empirical representation of population growth in the United States equation (iii) gives results of a very high degree of accuracy. Indeed, interpolation on this curve for inter-censal years may obviously be relied upon with a greater probability that the estimated figures approximate the unknown true facts than is afforded by any other estimating expedient hitherto applied to the known data. An indication of the general exactness of this curve (iii) for estimating future population by extrapolation may be got in the following way. Suppose a mathematician of the Civil War period had desired to estimate the population of the United States in 1910, and had fitted a curve of the type of (ii), by the method of least squares to the known data available

to him, namely, the census counts of 1790, to 1860, inclusive, he would have got this result:

$$y = 8{,}619{,}800 - 5{,}680{,}540x + 822{,}709x^2 + 16{,}987{,}200 \log x \qquad \text{(iv)}$$

If he had calculated from this equation the probable population in 1910, the figure he would have obtained would have been 92,523,000, a result only approximately a half million, or 0.6%, in error, as subsequent events proved. A prophecy less than 1% in error of an event to happen 50 years later is undeniably good predicting.

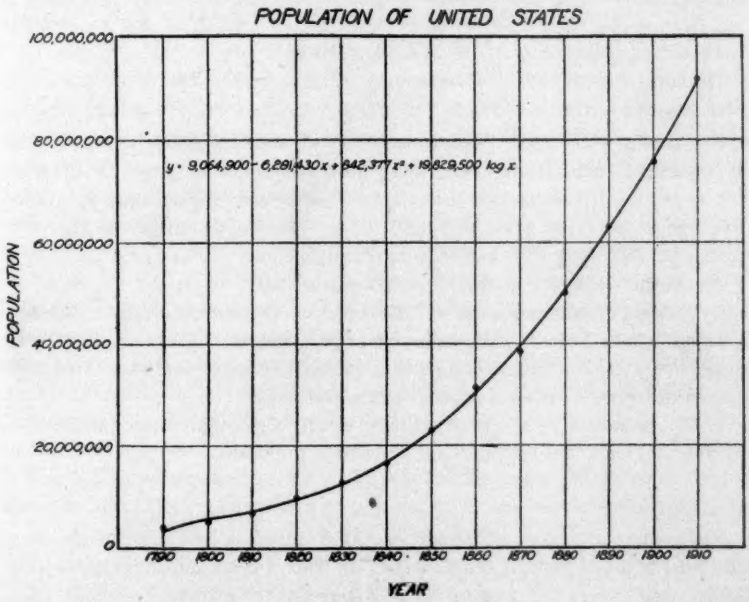

FIG. 1

Diagram showing observed and calculated populations (from logarithmic parabola) from 1790 to 1920.

It is of interest to exhibit the equations and results in predicting the 1910 population obtained by fitting our logarithmic parabola to the data available after the completion of each successive census from 1870 on. We have:

Data for 1790 to 1870, inclusive:

$$y = 8{,}287{,}700 - 5{,}300{,}270x + 795{,}540x^2 + 15{,}778{,}000 \log x \qquad \text{(v)}$$

Predicted population in 1910 = 91,201,000.

Deviation of prediction from actual, 1910 = − 771,000.

Percentage error = 0.8%.

Data for 1790 to 1880 inclusive:

$$y = 7{,}981{,}100 - 4{,}971{,}040x + 764{,}896x^2 + 14{,}993{,}500 \log x \qquad \text{(vi)}$$

Predicted population in 1910 = 89,128,000.
Deviation of prediction from actual, 1910 = − 2,844,000.
Percentage error = 3%.

Data for 1790 to 1890, inclusive:

$$y = 9{,}013{,}800 - 6{,}242{,}170x + 839{,}782x^2 + 19{,}744{,}300 \log x \quad \text{(vii)}$$

Predicted population in 1910 = 91,573,000.
Deviation of prediction from actual, 1910 = − 399,000.
Percentage error = 0.4 per cent.

Data for 1790 to 1900 inclusive:

$$y = 8{,}748{,}000 - 5{,}880{,}890x + 821{,}001x^2 + 18{,}232{,}100 \log x \quad \text{(viii)}$$

Predicted population in 1910 = 91,148,000.
Deviation of prediction from actual, 1910 = −824,000.
Percentage error = 0.9%.

Beginning with 1860 (equation (iv)) and coming down to 1900, our hypothetical statistician would have been only once in error as much as 1% in his prediction of the 1910 population by this logarithmic parabola. The one larger error is for the 1880 curve, where apparently the aberrant counts of 1860 and 1870 exert an undue influence.

Altogether it seems justifiable to conclude that:

1. A logarithmic parabola of the type of equation (ii) describes the changes which have occurred in the population of the United States in respect of its gross magnitude, with a higher degree of accuracy than any empirical formula hitherto applied to the purpose.

2. The accuracy of the graduation given by this logarithmic parabola is entirely sufficient for all practical statistical purposes.

II.

Satisfactory as the empirical equation above considered is from a practical point of view, it remains the fact that it is an empirical expression solely, and states no general law of population growth. Insofar it is obviously an undesirable point at which to leave the problem of the mathematical expression of the change of population in magnitude.

It is quite clear on *a priori* grounds, as was first pointed out by Malthus in non-mathematical terms, that in any restricted area, such as the United States, a time must eventually come when population will press so closely upon subsistence that its rate of increase per unit of time must be reduced to the vanishing point. In other words, a population curve may start, as does that shown in figure 1, with a convex face to the base, but presently it must develop a point of inflection, and from that point on present a concave face to the x axis, and finally become asymptotic, the asymptote representing the maximum number of people which can be supported on the given fixed area.[8] Now, while an equation like (ii) can, and will in due time, develop a point of inflection and become concave to the base it never can become asymptotic. It, therefore, cannot

be regarded as a hopeful line of approach to a true law of population growth.

What we want obviously is a mathematical picture of the *whole* course of population in this country. It is not enough to be able to predict twenty or fifty years ahead as our logarithmic parabola is able to do satisfactorily, *in one portion of the whole curve*. How absurd equation (iii) would be over a really long time range is shown if we attempt to calculate from it the probable population in, say, 3000 A.D. It gives a value of 11,822,000,000. But this is manifestly ridiculous; it would mean a population density of 6.2 persons per acre or 3968 persons per square mile.

It would be the height of presumption to attempt to predict *accurately* the population a thousand years hence. But any real law of population growth ought to give some general and approximate indication of the number of people who would be living at that time within the present area of the United States, provided no cataclysmic alteration of circumstances has in the meantime intervened.

It has seemed worth while to attempt to develop such a law, first by formulating a hypothesis which rigorously meets the logical requirements, and then by seeing whether in fact the hypothesis fits the known facts. The general biological hypothesis which we shall here test embodies as an essential feature the idea that the rate of population increase in a limited area at any instant of time is proportional (*a*) to the magnitude of the population existing at that instant (amount of increase already attained) and (*b*) to the still unutilized potentialities of population support existing in the limited area.

The following conditions should be fulfilled by any equation which is to describe adequately the growth of population in an area of fixed limits.

1. Asymptotic to a line $y = k$ when $x = +\infty$.
2. Asymptotic to a line $y = 0$ when $x = -\infty$.
3. A point of inflection at some point $x = \alpha$ and $y = \beta$.
4. Concave upwards to left of $x = \alpha$ and concave downward to right of $x = \alpha$.
5. No horizontal slope except at $x = \pm\infty$.
6. Values of y varying continuously from 0 to k as x varies from $-\infty$ to $+\infty$.

In these expressions y denotes population, and x denotes time.

An equation which fulfils these requirements is

$$y = \frac{be^{ax}}{1 + ce^{ax}} \qquad \text{(ix)}$$

when a, b and c have positive values.

In this equation the following relations hold:

$$x = +\infty \qquad y = \frac{b}{c} \qquad \text{(x)}$$

$$x = -\infty \qquad y = 0 \tag{xi}$$

Relations (x) and (xi) define the asymptotes.

The point of inflection is given by $1-ce^{ax} = 0$, or

$$x = -\frac{1}{a}\log c \qquad y = \frac{b}{2c} \tag{xii}$$

The slope at the point of inflection is $\frac{ab}{4c}$.

Expressing the first derivative of (xi) in terms of y, we have

$$\frac{dy}{dx} = \frac{ay(b-cy)}{b}. \tag{xiii}$$

Putting the equation in this form shows at once that it is identical

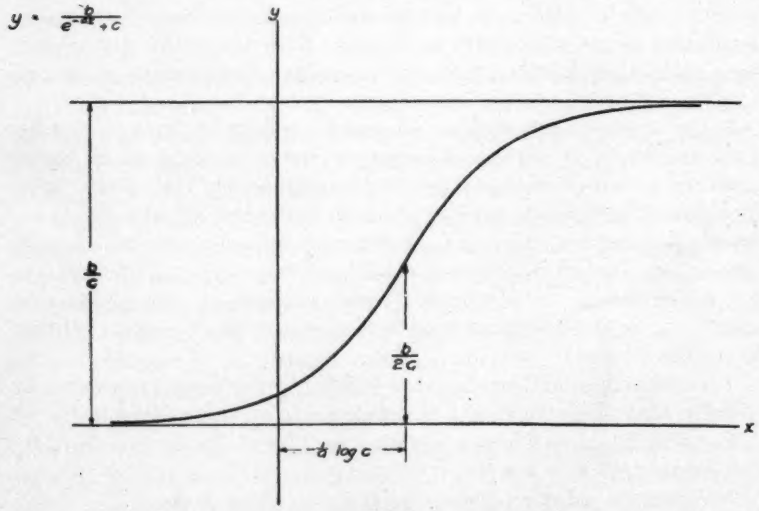

FIG. 2

General form of curve given by equation (ix).

with that describing an autocatalyzed chemical reaction, a point to which we shall return later.

The general form of the curve is shown in figure 2.

The question now is how well does (ix) represent the known historical facts as to the growth in population of the United States, and to what legitimate deductions as to the future course of population in this country does it lead?

It is obvious that equation (ix) as it stands cannot be fitted to observational data by the method of least squares. It is possible to write momental equations and fit by the method of moments, but at this tim we do not care to develop that method because, as will presently appear,

we do not regard equation (ix) as the final development of this type of equation for representing population, and we have no desire to encumber the literature with a mathematical discussion which we expect later to discard.

For present purposes it will be sufficient to fit (ix) to the observations by passing it through three points. Given three equally spaced ordinates, y_1, y_2 and y_3, the necessary equations are:

$$\frac{b}{c} = \frac{2y_1y_2y_3 - y_2^2(y_1 + y_3)}{y_1y_3 - y_2^2} \qquad \text{(xiv)}$$

$$a = \log_{10} \frac{y_2\left(\frac{b}{c} - y_1\right)}{y_1\left(\frac{b}{c} - y_2\right)} \div h \log_{10} e \qquad \text{(xv)}$$

where h is the abscissal distance in years between y_1 and y_2, or y_2 and y_3.

$$c = \frac{1}{y_2 - y}\left(\frac{y_1}{e^{a\alpha}} - \frac{y_2}{e^{a(\alpha+h)}}\right) \qquad \text{(xvi)}$$

where α is the abscissal distance in years from the origin to y_1.

Putting x_1 at 1790, x_2 at 1850, and x_3 at 1910, and taking origin at 1780 we have

$$\alpha = 10 \qquad y_1 = 3{,}929^9$$

$$h = 60 \qquad y_2 = 23{,}192$$

$$y_3 = 91{,}972$$

and taking (ix) in the form

$$y = \frac{b}{e^{-ax} + c}, \qquad \text{(xvii)}$$

we find these values for the constants:

$$y = \frac{2{,}930.3009}{e^{-.0313395x} + 0.014854} \qquad \text{(xviii)}$$

The closeness with which this curve fits the known facts is shown in table 3.

The closeness of fit of this curve is shown graphically in figure 3.

Though empirically arrived at this is a fairly good fit of theory to observations. The root-mean square error from the last column is 463,000, or slightly smaller than that from the logarithmic parabola in table 2. It must not be forgotten, however, that the root-mean square error is reduced in the present case by virtue of the fact that in three out of the 13 ordinates theory and observation are made, by the procrustean method of fitting, to coincide exactly. The most that can be asserted is that

TABLE 3

RESULTS OF FITTING POPULATION DATA 1790 TO 1910 BY EQUATION (XVIII)

YEAR	OBSERVED POPULATION	CALCULATED POPULATION BY EQUATION (XVIII)	ERROR
1790	3,929,000	3,929,000	0
1800	5,308,000	5,336,000	+ 28,000
1810	7,240,000	7,228,000	— 12,000
1820	9,638,000	9,757,000	+119,000
1830	12,866,000	13,109,000	+243,000
1840	17,069,000	17,506,000	+437,000
1850	23,192,000	23,192,000	0
1860	31,443,000	30,412,000	—1,031,000
1870	38,558,000	39,372,000	+ 814,000
1880	50,156,000	50,177,000	+ 21,000
1890	62,948,000	62,769,000	— 179,000
1900	75,995,000	76,870,000	+ 875,000
1910	91,972,000	91,972,000	0

equation (xviii) gives nearly or quite as good a fit to the observations as does the logarithmic parabola. If we properly graduated the data, by the method of moments, we should probably get a result measurably better than that from equation (iii).

The significance of the result lies in this consideration. A curve which on *a priori* grounds meets the conditions which must be satisfied by a true law of population growth, actually describes with a substantial degree of accuracy what is now known of the population history of this country.

Let us examine some further consequences which flow from equation (xviii). The first question which interests one is this: when did or will the population curve of this country pass the point of inflection, and exhibit a progressively diminishing instead of increasing rate of growth? From (xii) it is easily determined that this point occurred about *April 1, 1914*, on the assumption that the numerical values of (xviii) reliably represent the law of population growth in this country. In other words, so far as we may rely upon present numerical values, the United States has already passed its period of most rapid population growth, unless there comes into play some factor not now known and which has never operated during the past history of the country to make the rate of growth more rapid. This latter contingency is improbable. While prophecy is a dangerous pastime, we believe, from the fragmentary results already announced, that the 1920 census will confirm the result indicated by our curve, that the period of most rapid population growth was passed somewhere in the last decade. The population at the point of inflection works out to have been 98,637,000, which was in fact about the population of the country in 1914.

The upper asymptote given by (xviii) has the value 197,274,000 roughly. This means that according to equation (xviii) the maximum population which continental United States, as now areally limited, will ever have will be roughly twice the present population. We fear that some will condemn at once the whole theory because the magnitude of this number is not sufficiently imposing. It is so easy, and most writers on population have been so prone, to extrapolate population by geometric series, or by a parabola or some such purely empirical curve, and arrive at stupendous figures, that calm consideration of real probabilities is most difficult to obtain. While, as will appear from the next section of this

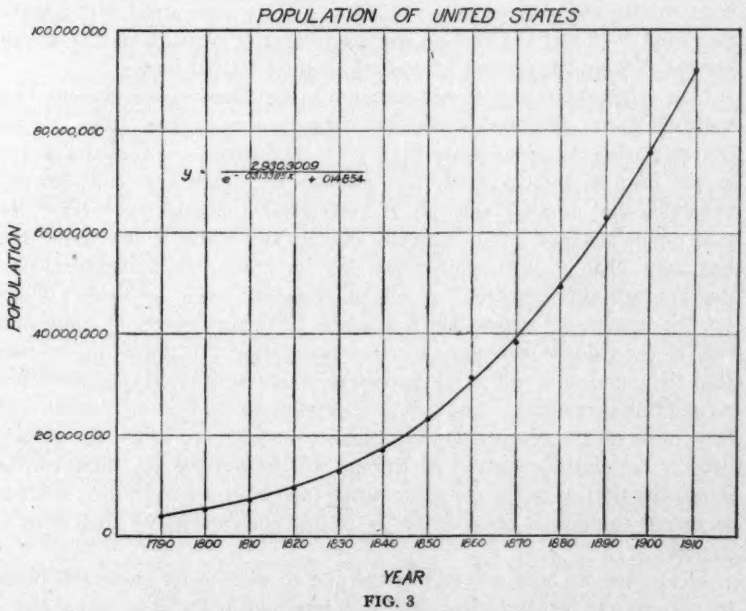

FIG. 3

Showing result of fitting equation (xviii) to population data.

paper, we have no desire to defend the numerical results of this section, and indeed ourselves regard them only as a rough first approximation, it remains a fact that if anyone will soberly think of every city, every village, every town in this country having its present population multiplied by 2, and will further think of twice as many persons on the land in agricultural pursuits, he will be bound, we think, to conclude that the country would be fairly densely populated. It would have about 66 persons per square mile of land area.

It will at once be pointed out that many European countries have a much greater density of population than 66 persons to the square mile,

as for example Belgium with 673, Netherlands with 499, etc. But it must not be forgotten that these countries are far from self-supporting in respect of physical means of subsistence. They are economically self-supporting, which is a very different thing, because by their industrial development at home and in their colonies they produce money enough to buy physical means of subsistence from less densely populated portions of the world. We can, of course, do the same thing, provided that by the time our population gets so dense as to make it necessary there still remain portions of the globe where food, clothing material, and fuel are produced in excess of the needs of their home population. But in this, and in any other scientific discussion of population, it is necessary to limit sharply the *area* one is to talk about. This paper deals with population, and by direct implication the production of physical means of subsistence, within the present area of continental United States.

Now 197,000,000 people will require, on the basis of our present food habits,[10] about 260,000,000,000,000 calories per annum. The United States, during the seven years 1911–1918, produced as an annual average, in the form of human food, *both primary and secondary* (i.e., broadly vegetable and animal), only 137,163,606,000,000 calories per year.[11] So that unless our food habits radically change, and a man is able to do with less than 3000 to 3500 calories per day, or unless our agricultural production radically increases,[12] it will be necessary when our modest figure for the asymptotic population is reached, to import nearly or quite one-half of the calories necessary for that population. It seems improbable that the population will go on increasing at any very rapid rate after such a condition is reached. And is it at all reasonable to suppose that at such time, with all the competition for means of subsistence which the already densely populated countries of Europe will be putting up, there can be found any portion of the globe producing food in excess of its own needs to an extent to make it possible for us to find the calories we shall need to import?

Altogether, we believe it will be the part of wisdom for anyone disposed to criticise out asymptotic value of a hundred and ninety-seven and a quarter millions because it is thought too small, to look further into all the relevant facts.

III

With the above numerical results in hand it is desirable to discuss a little further the general theory of population growth set forth in the preceding section. At the outstart let it be said that we are convinced that equation (ix) represents no more than a first approximation to a true law of population growth. There are several characteristics of this curve which are too rigid and inelastic to meet the requirements of such a law. In (ix) the point of inflection must of necessity lie exactly half-way between the two asymptotes. Furthermore the half of the curve lying

to the right of the point of inflection is an exact reversal of the half lying to the left of that point. This implies that the forces which during the latter part of the population history of an area act to inhibit the rate of population growth are equal in magnitude, and exactly similarly distributed in time, to the forces which in the first half of the history operate to accelerate growth. We do not believe that such rigid and inelastic postulates as these are, in fact, realized in population growth.

The same objections apply to the use of the equation of an autocatalytic reaction to the representation of organic growth in the individual. This fact has been noted by Robertson[13] who was the first to discover that, in general, growth follows much the same curve as that of autocatalysis. What needs to be done is to generalize (ix) in some such form as will free it from the two restrictive features (location of point of inflection and symmetry) we have mentioned, and will at the same time retain its other essential features. We are working along this line now and hope presently to reach a satisfactory solution.

We attach no particular significance to the *numerical* results of the preceding section. They obviously can give only the roughest approximation to probable future values of the population of the United States. Our only purpose in presenting them at all at this time is to demonstrate that the hypothesis here advanced as to the law of population growth, even when fitted by a rough and inadequate method, so closely describes the known facts regarding the past history of that growth, as to make it potentially profitable to continue the mathematical development and refinement of this hypothesis further. There is much that appeals to the reason in the hypothesis that growth of population is fundamentally a phenomenon like autocatalysis. In a new and thinly populated country the population already existing there, being impressed with the apparently boundless opportunities, tends to reproduce freely, to urge friends to come from older countries, and by the example of their well-being, actual or potential, to induce strangers to immigrate. As the population becomes more dense and passes into a phase where the still unutilized potentialties of subsistence, measured in terms of population, are measurably smaller than those which have already been utilized, all of these forces tending to the increase of population will become reduced.

[1] Papers from the Department of Biometry and Bital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 13.

[2] Cf. for a discussion on the relation of curve fitting to true organic laws of change, Pearl, R. "Some Recent Studies on Growth," *Amer. Nat.*, **43**, 1909 (302–316).

[3] Perrin, E., "On Some Dangers of Extrapolation," *Biometrika*, **3**, 1904 (99–103).

[4] Pritchett, A. S., "A Formula for Predicting the Population of the United States," *Quart. Publ. Amer. Statistical Assoc.*, **2**, 1891 (278–286).

[5] Pearl, R., "Variation and Differentiation in Ceratophyllum," *Carnegie Inst. Washington, Publ.* **58**, 1907 (136).

[6] Cf. the following papers:

Curtis, M. R., 1914, "A Biometrical Study of Egg Production in the Domestic Fowl. IV. Factors Influencing the Size, Shape, and Physical Constitution of Eggs," *Arch. Entwicklungsmech. Organ.*, B. **39,** Heft 2/3, pp. 217–327.

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[7] Donaldson, H. H., 1908, "A Comparison of the Albino Rat with Man in Respect to the Growth of the Brain and of the Spinal Cord," *J. Compar. Neurol. Psych.*, **18,** No. 4, pp. 345–389.

Donaldson, H. H., 1909, "On the Relation of the Body Length to the Body Weight and to the Weight of the Brain and of the Spinal Cord in the Albino Rat (*Mus norvegicus var. albus*)," *Ibid.*, **19,** No. 2, pp. 155–167.

Donaldson, H. H., 1910, "On the Percentage of Water in the Brain and in the Spinal Cord of the Albino Rat," *Ibid.*, **20,** No. 3, pp. 119–144.

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Donaldson and Hatai, Shinkishi, 1911, "A Comparison of the Norway Rat with the Albino Rat in Respect to Body Length, Brain Weight, Spinal Cord Weight, and the Percentage of Water in Both the Brain and the Spinal Cord," *J. Compar. Neurol. Psych.*, **21,** pp. 417–458.

Hatai, Shinkishi, 1909, "Note on the Formulas Used for Calculating the Weight of the Brain in the Albino Rats," *Ibid.*, **19,** No. 2, pp. 169–173.

Hatai, Shinkishi, 1911, "A Formula for Determining the Total Length of the Leopard Frog (*R. pipiens*) for a Given Body Weight," *Anat. Rec.*, **5,** No. 6, pp. 309–312.

Hatai, Shinkishi, 1911, "An Interpretation of Growth Curves from a Dynamical Standpoint," *Ibid.*, **5,** No. 8, pp. 373–382.

[8] Always, be it clearly understood, on the assumption that the average standard of living, method of agricultural production, etc., either do not further change at all in the period between the end of historical past record of fact and the time when dy/dx becomes negligibly small as the asymptote is approached, or that the net effective magnitude of any such changes as do occur will be relatively so small as to be negligible in comparison with the effect of such factors as reproduction and immigration in determining the relation between population and time in an area of fixed limits. In any mathematical treatment of the subject these factors of standard of living, methods of agriculture, etc., represent essentially constant (and hence omitted) parameters of any assumed functional relation between population and time.

[9] Omitting 000 here and in the subsequent calculations till the end.

[10] Cf. Pearl, R., *The Nation's Food*, Philadelphia (W. B. Saunders Company), 1920 (247).

[11] Pearl, R., *loc. cit.*, p. 76.

[12] As a matter of fact East, in his able presidential address on "Population," before the American Society of Naturalists has shown that the United States has already entered upon the era of diminishing returns in agriculture in this country.

[13] Robertson, T. Brailsford, "On the Normal Rate of Growth of an Individual and Its Biochemical Significance," *Arch. Entwickmech. Organ.*, **25,** pp. 581–614.

Robertson, T. Brailsford, "Further Remarks on the Normal Rate of Growth of an Individual, and Its Biochemical Significance," *Ibid.*, **26,** pp. 108–118.

ON THE SPRINGFIELD RIFLE AND THE LEDUC FORMULA*

BY ARTHUR GORDON WEBSTER

CLARK UNIVERSITY, WORCESTER, MASS.

Read before the Academy, April 26, 1920

One of the formulae of interior ballistics found most useful, at any rate by practical ballisticians in the United States, is the formula of Leduc, which says that the velocity of a shot in the bore of a gun is graphically represented in terms of the distance travelled by a rectangular hyperbola, giving the formula:

$$v = \frac{as}{b + s}$$

where v is the velocity, s the distance travelled by the shot.

The writer had the curiosity to try this formula on the Springfield rifle. The observations were made by the gauge described in these PROCEEDINGS, **5**, July, 1919 (259–263). The mode of deducing the Leduc formula is decidedly open to criticism since it assumes the combustion to be instantaneous, and the expansion adiabatic, neither of which is true and it is perfectly obvious that powders giving curves so different as those published in the paper quoted cannot possibly give v, s curves of the same shape. Nevertheless, it turns out that for the rifle the Leduc formula answers very well indeed.

Observations were made on the time of reaching different points in the barrel by Mr. H. C. Parker, assistant in the Ballistic Institute of Clark University. The method was to put a fine wire insulated with enamel down the barrel of the rifle to a certain distance. When the shot reached it a circuit was made which made a current passing through the oscillograph. The motion of the oscillograph was photographed on a rotating drum and thus the times were obtained. The curve obtained for the actual times of reaching ten or a dozen distances was exactly similar to that shown in the paper cited above, obtained by a double integration of the pressure-time curve. The velocities were obtained from Mr. Parker's curve by a graphic differentiation, and from the known velocities and distances the formula was obtained. Even without the method of least squares the formula $v = \frac{1133s}{19.8 + s}$ answers very well where the distances are given in centimeters and the velocities in meters per second.

* Contribution from the Ballistic Institute, No. 7.

ALKYL AMIDES OF ISOTHIOCYANACETIC ACID

By Treat B. Johnson, Arthur J. Hill and Erwin B. Kelsey

Department of Chemistry, Yale University

Communicated April 10, 1920

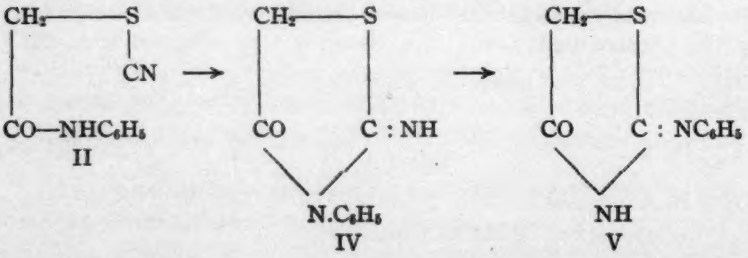
Theoretically, anilides of chloracetic acid, of which chloracetanilide I is the simplest representative, might be expected to interact with potassium thiocyanate in two ways, namely, to form either a normal thiocyanate or its structural isomer, an isothiocyanate, as is represented by formulas II and III, respectively. A careful study of this reaction, however, has revealed the fact that it is productive only of the thiocyanate

$$ClCH_2CONHC_6H_5$$
I

$$NCS.CH_2CONHC_6H_5 \qquad SCN.CH_2CONHC_6H_5$$
II III

form of these two isomers, or that combination in which sulphur is joined to carbon as represented in formula II. In every case, where the reaction has been represented as proceeding in the other manner, a careful examination of the properties of the reaction-product has revealed an incorret interpretation of the change, and shown that the primary reaction-product possesses a different constitution than that of an isothiocyanate. In other words, we may conclude from the evidence thus far presented, that no anilide of isothiocyanacetic acid has yet been prepared.[1]

Wheeler and Johnson[2] submitted this reaction between potassium thiocyanate and chloracetanilides to a very careful investigation and showed that the primary product of reaction is always a normal thiocyanate. They found, however, that these combinations are not always stable and in some cases it was extremely difficult to isolate them in a state of purity. This instability is explained by the fact that they easily undergo an unique rearrangement and are transformed into pseudothiohydantoins. In no case were normal thiohydantoins obtained by rearrangement. The isomerization of thiocyanacetanilide may be represented as follows:



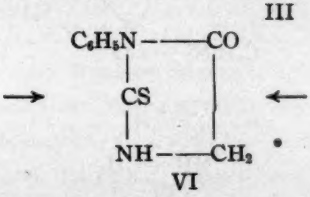
The conclusions recorded in a recent publication by Beckurts and Frerichs[3] have now forced us to turn our attention again to this field of

research. These investigators have included in their recent work several new anilides of chloracetic acid, and in the case of three of them they interpret the reaction with potassium thiocyanate as leading to the formation of isothiocyanates. The anilides which interact with potassium thiocyanate in this unique manner are the *m*-toluide, *p*-chloranilide and *p*-anisidide of chloracetic acid.

In the light of these interesting results, it has become necessary for us to take up again this question of constitution from an entirely new viewpoint, and apply the synthetical method of investigation. The question of tautomeric behavior of potassium thiocyanate must be excluded from further consideration, and a fundamental method of synthesis be developed whereby only true isothiocyanate combinations corresponding to formula III can be formed. At present such a method of synthesis is not known. These isothiocyanates if formed would not revert by rearrangement to normal thiocyanates, and should exhibit a chemical behavior quite as characteristic as that of the isomeric thiocyanates. Theoretically combinations corresponding to formula III should easily undergo isomerization with formation of normal thiohydantoins. In other words, if the isothiocyanate III is a product of reaction when potassium thiocyanate interacts with chloracetanilide as Frerich and Beckurts assume, it should act in an entirely different manner than the thiocyanate II and isomerize to the same hydantoin VI, as is obtained by interaction of phenylisothiocyanate with glycine[4] or its ethyl ester. These changes are represented as follows:

$$ClCH_2CONHC_6H_5 \longrightarrow SCN.CH_2CONHC_6H_5$$

III



VI

$$C_6H_5NCS + NH_2CH_2COOH \longrightarrow C_6H_5NHCSNHCH_23COOH$$

We are now in position to report a practical method of synthesis which enables us to obtain anilides of isothiocyanacetic acid. Starting with chloracetanilide for illustration the complete operation may be outlined as follows: Chloracetanilide is first converted into the anilide of glycocoll (glycine) VII, and this then combined with carbonbisulphide when the corresponding dithiocarbamate VIII is obtained in excellent yield. This salt VIII is a crystalline substance melting at 145° and its formation from chloracetanilide is expressed as follows:

$$ClCH_2CONHC_6H_5 + 2NH_3 = NH_4Cl + NH_2CH_2CONHC_6H_5$$

VII

$2NH_2CH_2CONHC_6H_5 + CS_2 = C_6H_5NH.COCH_2NHCSSH.NH_2CH_2CO\text{-}NHC_6H_5$

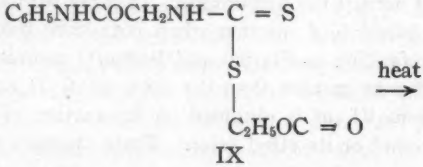
VIII

The dithiocarbamate VIII interacts smoothly with ethyl chlorformate to give the corresponding ester represented by formula IX. This ester IX is the first representative of its type to be described in the chemical

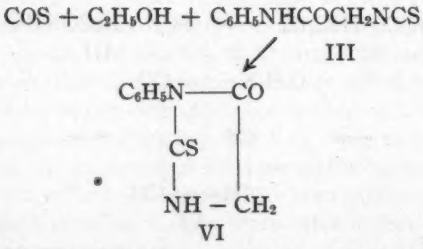
$C_6H_5NHCOCH_2NH.CSSH.NH_2CH_2CONHC_6H_5 + Cl.COOC_2H_5 =$
$C_6H_5NHCOCH_2NH.CSS.COOC_2H_5 + C_6H_5NH.CO.CH_2NH_2HCl$

IX

literature, and is a crystalline substance melting at 100°. This ester can be crystallized without decomposition, but undergoes the normal decomposition, characteristic of its constitution, when heated above its melting point temperature. It then breaks down into carbonoxysulphide, ethyl alcohol and the unknown isothiocyanate III. This characteristic decomposition reaction is expressed as follows:



IX



VI

Thus far we have not succeeded in stopping our reaction at the stage where the isothiocyanate III is formed. To our surprise this substance is so unstable that the heat applied to decompose the dithiocarbamic ester IX is sufficient to produce immediate rearrangement of the resulting isothiocyanate III into the normal phenylthiohydantoin VI. The yield of this cyclic compound is excellent, and as we anticipated it proved to be identical with the thiohydantoin previously described in the literature.[5] We obtained no evidence of the formation of either of the two pseudothiohydantoins IV and V which result by rearrangement of thiocyanacetanilide II. In other words, our method of synthesis not only excludes the possibility of formation of a normal thiocyanate, but has also revealed

to us for the first time the striking dissimilarity in chemical behavior between these two types of isomeric compounds.

It seems safe to conclude, therefore, from the results of this new work that any isothiocyanate formed by interaction of potassium thiocyanate with a secondary chloracetanilide will be found to be unstable and easily transformed into its corresponding thiohydantoin. Whether Beckurts and Frerichs were actually dealing in their work with such normal thiohydantoins instead of the pseudo modifications must be decided by further investigation. In the case of tertiary anilides of isothiocyanacetic $SCN.CH_2CO.NR_2$, such cyclic rearrangements cannot take place, and in such cases we expect to obtain acyclic combinations containing the grouping NCS functioning as a true isothiocyanate. Investigations dealing with various phases of this interesting problem are now in progress and the results will be published in the *Journal of the American Chemical Society*.

[1] Wheeler and Merriam, *J. Amer. Chem. Soc.*, **23**, 1901 (283).

[2] *Amer. Chem. J.*, **28**, 1902 (121); Johnson, *J. Amer. Chem. Soc.*, **25**, 1903 (483).

[3] *Arch. Pharm.*, **253** (233); *Chem. Zentrbl.*, **2**, 1915 (614); *Chem. Abstracts*, **10**, 1916, (888).

[4] Aschan, *Ber. chem. Ges.*, **17**, 1884 (420); Marckwald, Neumark and Stelzner, *Ibid.*, **24**, 1891 (3278).

[5] *Loc. cit.*

STUDIES OF MAGNITUDES IN STAR CLUSTERS. XI. FREQUENCY OF CURVES THE ABSOLUTE MAGNITUDE AND COLOR INDEX FOR 1152 GIANT STARS.

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Communicated by G. E. Hale, April 8, 1920

Kapteyn, Schwarzschild, Parkhurst, Russell, and other investigators of statistical problems in stellar astronomy have paid special attention to the relative frequency of successive values of the absolute brightness of stars (the luminosity law) and the relative frequency of different spectral or color types. The laws of the frequencies of absolute magnitude and spectrum are indeed fundamental in studies of stellar evolution and the arrangement of stars in space; but to determine these laws, at the same time keeping the errors due to unavoidable and vitiating selection of data at a minimum, is by no means a simple process. Insufficient knowledge of stellar distances, and frequently of apparent magnitudes and spectra as well, presents a serious obstacle. In particular, it is difficult to obtain satisfactory luminosity curves for each spectral type, or representative spectral curves for small and clearly defined intervals of absolute magnitude. The luminosity curve that does not differentiate

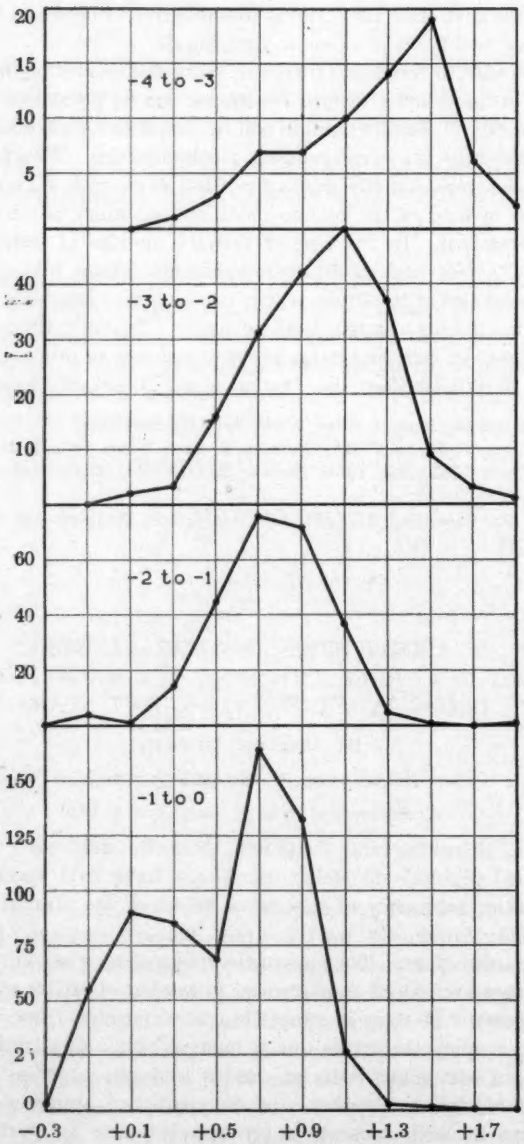

FIG. 1

Curves of the frequency of color index for successive intervals of absolute magnitude. Ordinates are numbers of stars; abscissae are color indices.

spectral types conceals the important differences for red and blue stars of the giant-dwarf phenomena; and the spectral (or color) curve that includes a great range of absolute magnitude is of limited value because

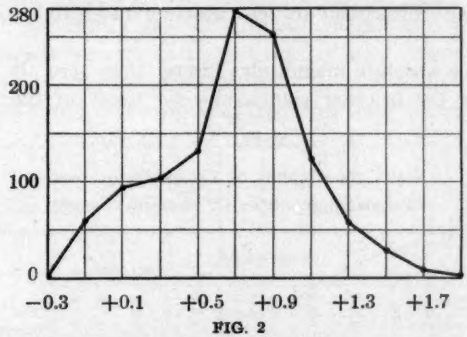

FIG. 2

Integrated color curve for 1145 giant stars of all absolute magnitudes brighter than zero. Coördinates as in Fig. 1.

it integrates indiscriminately the various masses, densities, ages, and other factors that appear to change with luminosity.

The study of magnitudes and colors in star clusters affords, however, for the brightest of giant stars, a fairly accurate determination of the frequencies of magnitudes and colors for small intervals of color and

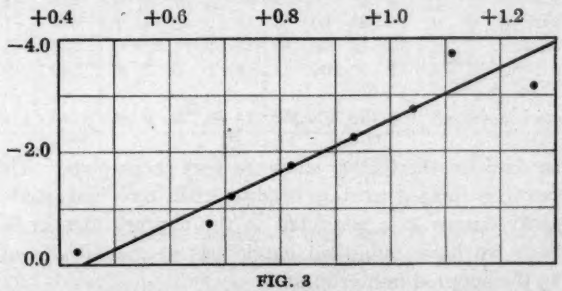

FIG. 3

Change of mean color index with brightness for 1143 giant stars in clusters. Ordinates are absolute photovisual magnitudes; abscissae are color indices. (See last three columns of Table I.)

magnitude, respectively. The relative apparent magnitudes and distances, which in most cases are prerequisite to a knowledge of absolute brightness, are sufficiently known in nine systems to permit a combination of all observations for a statistical investigation of the above-mentioned astrophysical laws.

The discussion of the observational data for 1152 stars brighter than absolute photovisual magnitude zero is summarized in the tabular results and illustrative curves of the present note.

The clusters whose stars are included in this synopsis of absolute magnitude are Messier 3, 5, 11, 13, 15, 30, 68; N. G. C. 4147, 7006. Their distances range between seven and seventy thousand parsecs; the apparent magnitudes that correspond to zero absolute magnitude range between 14.3 and 19.2.

The available absolute magnitudes fainter than zero are less accurate than those for the brighter giants, and for many of the most distant

TABLE I

DATA FOR CURVES OF COLOR FREQUENCY

(Tabulated quantities are numbers of stars)

ABSOLUTE PHOTOVISUAL MAGNITUDE	COLOR INDEX												ALL COLORS	MEAN MAGNITUDE	MEAN COLOR INDEX
	−0.4 to −0.2	−0.2 to 0.0	0.0 to +0.2	+0.2 to +0.4	+0.4 to +0.6	+0.6 to +0.8	+0.8 to +1.0	+1.0 to +1.2	+1.2 to +1.4	+1.4 to +1.6	+1.6 to +1.8	+1.8 to +2.0			
−5.0 to −4.5	0	0	0	0	0	0	0	0	0	0	0	0	0		
−4.5 to −4.0	0	0	0	1	0	0	0	1	0	0	0	0	2	−4.38	+0.77
−4.0 to −3.5	0	0	0	1	2	2	2	3	3	6	1	0	20	−3.74	+1.11
−3.5 to −3.0	0	0	0	0	1	5	5	7	11	13	5	2	49	−3.18	+1.26
−3.0 to −2.5	0	0	1	1	7	14	13	21	25	6	1	1	90	−2.77	+1.04
−2.5 to −2.0	0	0	1	2	9	17	29	29	12	1	2	0	102	−2.25	+0.93
−2.0 to −1.5	0	0	1	4	22	16	39	24	3	1	0	0	110	−1.76	+0.82
−1.5 to −1.0	0	4	0	10	23	60	33	13	2	0	0	1	146	−1.22	+0.71
−1.0 to −0.5	1	1	16	34	18	60	77	18	1	1	0	0	227	−0.72	+0.67
−0.5 to 0.0	2	54	74	51	50	104	55	8	1	0	0	0	399	−0.23	+0.43
All magnitudes	3	59	93	104	132	278	253	124	58	28	9	4	1145	−1.18	+0.69

clusters the data for the fainter stars are very incomplete. The present study therefore is limited to stars brighter than zero magnitude, and for the luminosity curves it is restricted to the clusters Messier 3, 11, and 13, for which we have published catalogues of magnitudes and colors complete to the adopted fainter limit.

For intervals of 0.2 mag. in color index and of 0.5 mag. in absolute photovisual magnitude table I contains the number of stars from all nine clusters. Only seven of the 1152 stars fall outside the limits of this table. Their absolute magnitude and color indices are:

−5.45	+1.01	−2.77	+2.06
−4.14	+1.23	−2.19	−0.52
−5.09	+1.00	−1.03	−0.60
−5.01	+1.83		

It should be especially noted that only six stars (four of those above and two in table I) are brighter than -4.0, and the brightest recorded star is fainter than -5.5. We are able, moreover, to go much farther with this inference relative to an upper limit for stellar luminosity. The investigations carried on at Mount Wilson during the last six years have yielded provisional but sufficiently reliable absolute magnitudes in more than fifty clusters; they permit the statement that, *of more than a million stars, less than a tenth of 1% are brighter than -4, photovisually.* Since the brightest giants are red, the corresponding upper limit for absolute photo-

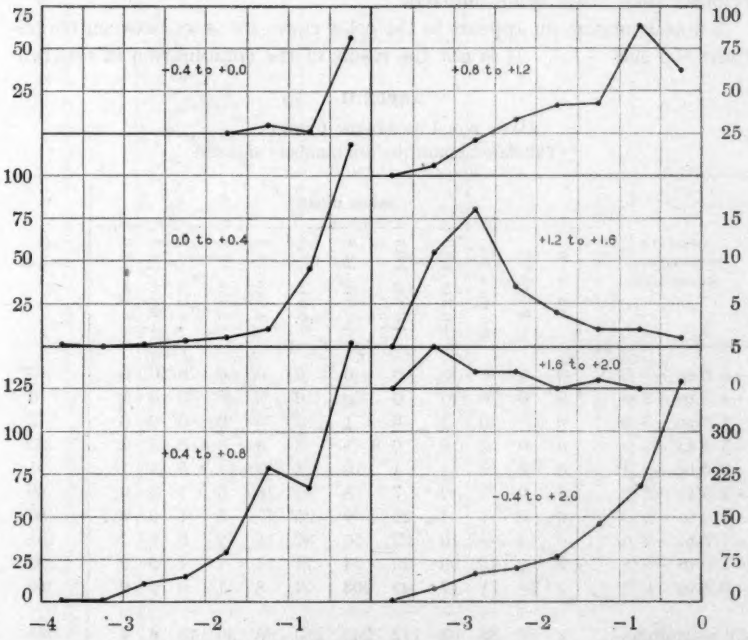

FIG. 4

Luminosity curves for different intervals of color index (Table II). Ordinates are numbers of stars; abscissae are absolute photovisual magnitudes.

graphic magnitude may be placed at -3. There appears to be scarcely a single star brighter than photovisual magnitude -6.

The existence of a maximum luminosity, indicated by these results, is probably closely related to the upper limits of stellar mass suggested by Eddington's researches on giant stars; it is highly significant in considerations of external galactic systems.

It is also of interest that of the 1152 stars only three may be considered slightly abnormal in color, in that they fall outside the limits of table I. This speaks for the uniformity of stars throughout the universe, and

for the comparability of the spectral phenomena in globular clusters with those of the solar neighborhood.

Color frequency-curves for intervals of one magnitude are shown in figures 1 and 2, based on the data of table I. The striking progression of maximum frequency toward the blue, with decreasing magnitude, again emphasizes the law of average color for giant stars in clusters, viz., *the brighter the giant star the greater its color index*. The progression of average color with luminosity is also clearly shown by the means in the last two columns of table I, which are plotted in figure 3, the six stars brighter than -4.0 being omitted.

A double maximum appears in the color curve for stars between magnitudes 0.0 and -1.0. It is not the result of the combination of observa-

TABLE II

DATA FOR LUMINOSITY CURVES
(Tabulated quantities are numbers of stars)

ABSOLUTE PHOTOVISUAL MAGNITUDE	COLOR INDEX												ALL COLORS
	−0.4 to −0.2	−0.2 to 0.0	0.0 to +0.2	+0.2 to +0.4	+0.4 to +0.6	+0.6 to +0.8	+0.8 to +1.0	+1.0 to +1.2	+1.2 to +1.4	+1.4 to +1.6	+1.6 to +1.8	+1.8 to +2.0	
−5.0 to −4.5	0	0	0	0	0	0	0	0	0	0	0	0	0
−4.5 to −4.0	0	0	0	0	0	0	0	0	0	0	0	0	0
−4.0 to −3.5	0	0	0	1	0	1	0	0	0	0	0	0	2
−3.5 to −3.0	0	0	0	0	0	1	3	3	6	5	3	2	23
−3.0 to −2.5	0	0	0	1	1	10	8	13	11	5	1	1	51
−2.5 to −2.0	0	0	1	2	7	8	17	16	6	1	2	0	60
−2.0 to −1.5	0	0	1	4	20	9	25	17	3	1	0	0	80
−1.5 to −1.0	0	4	0	10	22	56	32	11	2	0	0	1	138
−1.0 to −0.5	0	1	15	30	13	54	71	17	1	1	0	0	203
−0.5 to 0.0	2	54	71	47	49	103	54	8	1	0	0	0	389
All magnitudes	2	59	88	95	112	242	210	85	30	13	6	4	946

tions from clusters at different stages of development, because it is shown separately for Messier 3 and 13. It may be due to unequal evolutionary values for the different intervals of color—to a relatively speedier development, that is, throughout the interval of color index $+0.2$ to $+0.6$, for stars of a certain mass.

Table II, similar in form to table I, contains data for the luminosity curves (fig. 4), showing the distribution in color and absolute magnitude of 946 stars from Messier 3, 11, and 13. These curves, for intervals of 0.4 mag. in color index, indicate that our material gives the complete distribution of luminosity for the red giants, but that for the blue stars the maximum frequency falls below zero absolute magnitude.

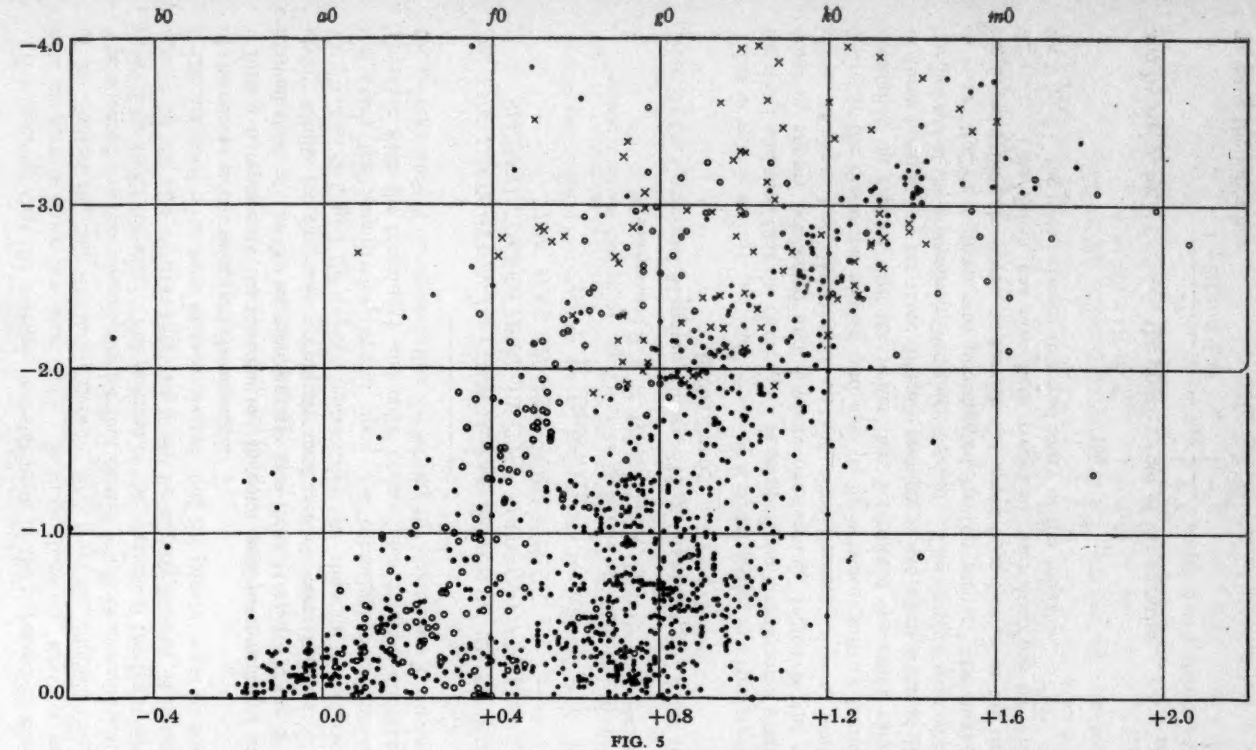

FIG. 5

Distribution of giant stars in color and brightness. Ordinates are absolute photovisual magnitudes; abscissae are color indices with the corresponding color class indicated at the top of the diagram. (See last paragraph of text for further explanation.)

It is believed that the average probable error of the paralaxes of these nine clusters is much less than 15%. Suppose, however, that for one of the clusters the true distance differs by 30% from the adopted value. The adopted absolute magnitudes for that cluster would be systematically in error by about 0.6 mag. The position of the maxima of the luminosity curves for the nine clusters together would be slightly altered, therefore, if the supposed error were uncompensated; but the general form of none of the curves would be materially changed.

Figure 5 represents the distribution in color and luminosity of the individual stars, the circles representing data from Messier 11, the dots giving results for the most accurately measured clusters (Messier 3, 5, 13, 15, and 68), and the crosses representing the stars for the clusters for which the magnitudes depend upon less extensive investigations. Together with the luminosity and color curves, this diagram illustrates the present state of our information concerning the giant stars in clusters.

ON THE DISTORTION IN CONFORMAL MAPPING WHEN THE SECOND COEFFICIENT IN THE MAPPING FUNCTION HAS AN ASSIGNED VALUE

By T. H. Gronwall

Technical Staff, Office of the Chief of Ordnance, Washington, D. C.

Communicated by E. H. Moore, April 27, 1920

Note III On Conformal Mapping Under Aid of Grant No. 207 From the Bache Fund

Let $w = z + a_2z^2 + \ldots + a_nz^n + \ldots$ be a power series in z convergent for $|z|<1$ and such that the circle $|z|<1$ is mapped conformally on a simple (that is, simply connected and nowhere overlapping) region in the w-plane. Koebe[1] has shown that on the circumference $|z| = r$, where $0<r<1$, the distortion $|dw/dz|$ and also $|z|$ lie between positive bounds depending on r alone, and the writer[2] has determined the exact values of these bounds. A far more difficult problem arises when some of the coefficients of the power series are given *a priori*. The simplest case where $a_2 = ae^{\gamma i}$ $(a \geq 0)$ is given was investigated by the writer,[3] the method employed failing, however, to furnish the upper bound of $|z|$ in the case $0 \leq a < 1$. This defect has now been remedied, and denoting by $r(a)$, for $0 \leq a < 1$, the root between zero and unity of the equation

$$\frac{2r}{1 + 2(a-1)r + r^2} - \log\frac{1+r}{1-r} = 0,$$

and by $\cos\beta$, for $0 < r \leq r(a)$, the positive root of the equation

$$\frac{2r}{1-2r\cos\beta + r^2} - \log\frac{1+r}{1-r} = 0,$$

we have the following:

THEOREM: *When the analytic function*

$$w = z + ae^{\gamma i}z^2 + a_3z^3 + \ldots + a_nz^n + \ldots$$

(where $a \geqq 0$ and γ real) maps the circle $|z|<1$ on the interior of a simple region D in the w-plane, then $a \leqq 2$, and we have, for $|z| = r$ and $0<r<1$,

$$\frac{1-r^2}{(1+ar+r^2)^2} < \left|\frac{dw}{dz}\right| < \begin{cases} \dfrac{1-2r\cos\beta+r^2}{(1-r)^{2+\cos\beta+a}(1+r)^{2-\cos\beta-a}} \\ [\text{for } a<1 \text{ and } r \leqq r(a)], \\ \dfrac{1+2(a-1)r+r^2}{(1-r)^3(1+r)} \\ [\text{for } a<1 \text{ and } r \geqq r(a), \text{ and for } 1 \leqq a \leqq 2], \end{cases}$$

$$\frac{r}{1+ar+r^2} < |w| < \begin{cases} \dfrac{1}{4(1+a)}\left[\left(\dfrac{1+r}{1-r}\right)^{1+a}-1\right]+\dfrac{1}{4(1-a)}\left[1-\left(\dfrac{1-r}{1+r}\right)^{1-a}\right] \\ [\text{for } a<1], \\ \dfrac{2-a}{4}\log\dfrac{1+r}{1-r}+\dfrac{a}{2}\dfrac{r}{(1-r)^2} \\ [\text{for } 1 \leqq a \leqq 2], \end{cases}$$

except that these bounds are reached for the functions w obtained upon replacing r by $ze^{\gamma i}$ and $-ze^{\gamma i}$ in the upper and lower bounds, respectively. When the region D is CONVEX, *then $a \leqq 1$, and we have*

$$\frac{1}{1+2ar+r^2} < \left|\frac{dw}{dz}\right| < \frac{1}{(1+r)^{1+a}(1+r)^{1-a}},$$

$$\left.\begin{array}{l} \dfrac{1}{\sqrt{1-a^2}}\arctan\dfrac{\sqrt{1-a^2}\,.\,r}{1+ar} \\ [\text{for } a<1], \\ \dfrac{r}{1-r} \\ [\text{for } a=1], \end{array}\right\} < |w| < \begin{cases} \dfrac{1}{2}\log\dfrac{1+r}{1-r} \\ [\text{for } a=0], \\ \dfrac{1}{2a}\left[\left(\dfrac{1+r}{1-r}\right)^a-1\right] \\ [\text{for } 0<a \leqq 1], \end{cases}$$

except for the functions w obtained as above.

As an application, let us suppose that the function

$$w = 1/z + c_1z + \ldots + c_nz^n + \ldots$$

(without constant term) maps $|z|<1$ on a simple region in the w-plane (this region containing, of course, $w = \infty$ in its interior). Inequalities of the form $|w|<k/r$ for $|z| = r$, $0<r<1$, k = constant, were obtained by Fricke[4] and the writer.[5] From the assumption, it follows that

$$W = 1/w = z + a_2z^2 + \ldots + a_nz^n + \ldots$$

maps $|z|<1$ on a simple region in the W-plane, and the application of our theorem with $a = 0$ gives

$$\frac{r}{1+r^2} < |W| < \frac{r}{1-r^2},$$

or

$$\frac{1}{r} - r < |w| < \frac{1}{r} + r$$

for $|z| = r$, $0 < r < 1$, except when

$$w = \frac{1}{z} + e^{2\alpha i} z$$

with α real, the upper and lower bounds of $|w|$ being then reached when $z = \pm e^{-\alpha i} r$ and $z = \pm i e^{-\alpha i} r$, respectively. This particular function maps $|z| < 1$ on the w-plane cut along the straight line from $w = -2e^{\alpha i}$ to $w = 2e^{\alpha i}$.

[1] Koebe, *Göttingen, Nachr. Ges. Wiss.*, 1909 (73).

[2] Gronwall, *Paris, C. R. Acad. Sci.*, **162**, 1916 (249).

[3] Gronwall, *Ibid.*, **162**, 1916 (316).

[4] Fricke and Klein, *Vorlesungen über die Theorie der automorphen Functionen*, II, Leipzig, 1912 (497).

[5] Gronwall, *Annals Math., Princeton*, Ser. 2, **16**, 1914 (74).

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*ON THE CONNECTION OF THE SPECIFIC HEATS WITH THE EQUATION OF STATE OF A GAS**

BY ARTHUR GORDON WEBSTER

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Read before the Academy, April 26, 1920

One would suppose that this subject was already exhausted, but I am disposed to believe that such is not the case. In these PROCEEDINGS, **5**, July, 1919 (286–288) I published a paper "On the Possible Form of the Equation of State of Powder Gases." In a private letter from M. LeChatelier, in commenting on this paper, he says: "Votre objection au sujet de nos chaleurs spécifiques nous était venue à l'esprit, mais nous pensions avoir démontré, en partant des principes de la thermodynamique, que toutes les fois qu'un fluide obéit a une équation caracteristique de la forme

$$V = F(P/T)$$

les chaleurs spécifiques sont indépendantes de la pression."

I do not find where, if anywhere, M. LeChatelier has published this conclusion. Certainly it is not in the great paper by Mallard and LeChatelier, *Journal de Physique*, Ser. 2, **4** (59–84), "Recherches sur la Combustion des Mélanges Gazeux Explosifs," where the matter is not mentioned. In a paper published in the *Physical Review*, **29**, No. 3, September, 1909, "On the Definition of an Ideal Gas," written by me, but suggested by my then colleague Professor Rosanoff, and published under our joint names, I have investigated the properties of a gas having the equation of state mentioned by M. LeChatelier, but have said nothing about the specific heats. Inasmuch as the matter seems to be not quite

* Contribution from the Ballistic Institute, Clark University, No. 8.

clear and inasmuch as in the five-foot shelf of treatises on thermodynamics in my library I cannot find the answer to the question that I have in mind, I think it worth while to take the matter up *ab initio*. To be sure my colleague Professor Charles A. Kraus has pointed out to me that van der Waals, in his excellent Lehrbuch der Thermodynamik, Kap. II, has partially treated the subject, and Professor Gilbert N. Lewis, whom I met yesterday on the Avenue, referred me to the same "well-known formula," and intimated that there was nothing more to be added. I am not quite convinced. After teaching thermodynamics for thirty years I find it is only now that my ideas on some points are becoming clarified.

Let us suppose any system to be specified by two coördinates, x and y, in terms of which all thermal data may be expressed, neither being the temperature. If we deal with unit of mass of a substance, the specific heat at constant x is defined by the equation

$$dQ_x = C_x dT_x,$$

where dT_x is the change of temperature made when x is constant and dQ is the amount of heat required for a change of state. Similarly for the specific heat at constant y:

$$dQ_y = C_y dT_y,$$

Now we have in any change

$$dT = \frac{\partial T}{\partial x}\,dx + \frac{\partial T}{\partial y}\,dy,$$

and consequently:

$$dQ_x = C_x \frac{\partial T}{\partial y}, \quad dQ_y = C_y \frac{\partial T}{\partial x} dx.$$

Since for an infinitesimal change the total amount of heat required is the sum of the two preceding, we have for any change whatever

$$dQ = C_y \frac{\partial T}{\partial x} dx + C_x \frac{\partial T}{\partial y}\,dy.$$

But, by the first law of thermodynamics, when we are dealing with a system under a uniform normal pressure p we have

$$dQ = dU + p dv$$

where U is the intrinsic energy of the substance. Writing out the perfect differentials dU and dv in terms of dx and dy, which are independent, we have the two partial differential equations:

$$\frac{\partial U}{\partial x} = C_y \frac{\partial T}{\partial x} - p \frac{\partial v}{\partial x},$$

$$\frac{\partial U}{\partial y} = C_x \frac{\partial T}{\partial y} - p \frac{\partial v}{\partial y}.$$

From these, eliminating U by cross differentiation, we get:

$$\frac{\partial C_x}{\partial x}\frac{\partial T}{\partial y} - \frac{\partial C_y}{\partial y}\frac{\partial T}{\partial x} + (C_x - C_y)\frac{\partial^2 T}{\partial x \partial y} = \frac{\partial(p,v)}{\partial(x,y)} \qquad (1)$$

From the second law of thermodynamics, introducing the integrating factor $1/T$ and the entropy S, we have

$$\frac{dQ}{T} = dS = \frac{\partial S}{\partial x}dx + \frac{\partial S}{\partial y}dy = \frac{C_y}{T}\frac{\partial T}{\partial x}dx + \frac{C_x}{T}\frac{\partial T}{\partial y}dy$$

from which we get in a similar manner:

$$\frac{\partial C_x}{\partial x}\frac{\partial T}{\partial y} - \frac{\partial C_y}{\partial y}\frac{\partial T}{\partial x} + (C_x - C_y)\frac{\partial^2 T}{\partial x \partial y} = \frac{(C_x - C_y)}{T}\frac{\partial T}{\partial x}\frac{\partial T}{\partial y} \qquad (2)$$

By a combination of these two equations we get

$$(C_x - C_y)\frac{\partial T}{\partial x}\frac{\partial T}{\partial y} = T\frac{\partial(p,v)}{\partial(x,y)}$$

which is equation (3) of my paper quoted above.

These are all the equations that we can get from the laws of thermodynamics and since any function has two partial derivatives for C_x and C_y, there will be four, which to be sure are connected by two equations. It is, I think, obvious that two equations are not sufficient. At any rate, let us proceed to an examination of the equation of state suggested by M. LeChatelier. If we have:

$$x = p,\ y = v, \qquad T = pf(v) = xf(y),$$

we at once obtain

$$\frac{\partial T}{\partial x} = f(y), \quad \frac{\partial T}{\partial y} = xf'(y), \quad \frac{\partial^2 T}{\partial x \partial y} = f'(y)$$

And accordingly in the equation of the first law we have

$$xf'(y)\frac{\partial C_x}{\partial x} - f(y)\frac{\partial C_y}{\partial y} + (C_x - C_y)f'(y) = 1. \qquad (3)$$

and in the equation of the second law

$$xf'(y)\frac{\partial C_x}{\partial x} - f(y)\frac{\partial C_y}{\partial y} + (C_x - C_y)f'(y) = (C_x - C_y)f'(y). \qquad (4)$$

We accordingly obtain the relations between the two specific heats:

$$xf'(y)\frac{\partial C_x}{\partial x} - f(y)\frac{\partial C_y}{\partial y} = 0, \qquad (5)$$

$$C_x - C_y = \frac{1}{f'(y)}. \qquad (6)$$

From (6) we get

$$\frac{\partial C_y}{\partial y} = \frac{\partial C_x}{\partial y} + \frac{f''}{f'^2}. \qquad (7)$$

Substituting this value in equation (5) we obtain,

$$x\frac{f'}{f}\frac{\partial C_x}{\partial x} - \frac{\partial C_x}{\partial y} = \frac{f''}{f'^2}, \tag{8}$$

a partial differential equation of the first order for C_x, which is of the form

$$P\frac{\partial z}{\partial x} + Q\frac{\partial z}{\partial y} = R$$

if we put,

$$C_x = z, \quad P = x\frac{d\log f(y)}{dy}, \quad Q = -1, \quad R = -\frac{d}{dy}\left(\frac{1}{f'}\right).$$

This is immediately integrated by Lagrange's method, if we write

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}, \quad \frac{dx}{x\dfrac{d\log f}{dy}} = -dy = -\frac{dz}{\dfrac{d}{dy}\left(\dfrac{1}{f'}\right)}.$$

We obtain two particular integrals, in which we are to put

$$u \equiv xf = T = \text{const.}, \quad v \equiv z - \frac{1}{f'} = \text{const.}, \quad v = \varphi(u),$$

where φ is an arbitrary function. We thus obtain finally

$$C_p = 1/f'(v) + \varphi(T), \quad C_v = \varphi(T) \tag{9}$$

and we find that neither of the specific heats is constant or independent of the pressure, nor is their difference constant, although it is independent of the pressure. Such a gas has no cohesion pressure, but does not in general have a zero Joule-Kelvin effect. If, on the contrary, we take a gas subject to the equation,

$$T = vg(p)$$

which has a zero Joule-Kelvin effect, we have a similar conclusion, interchanging p and v.

If, on the other hand, one of our variables is the absolute temperature, we are to write,

$$dQ = C_x dT + L_x dx \tag{10}$$

where L_x is the latent heat for constant x. We then obtain from the first law

$$\frac{\partial U}{\partial x} = L_x - p\frac{\partial v}{\partial x}, \quad \frac{\partial U}{\partial T} = C_x - p\frac{\partial v}{\partial T},$$

$$\frac{\partial C_x}{\partial x} - \frac{\partial L_x}{\partial T} = \frac{\partial(p, v)}{\partial(x, T)}, \tag{11}$$

and from the second,

$$\frac{\partial S}{\partial x} = \frac{L_x}{T}, \quad \frac{dS}{dT} = \frac{C_x}{T}$$

$$\frac{\partial C_x}{\partial x} - \frac{\partial L_x}{\partial T} = -\frac{L_x}{T}, \tag{12}$$

and combining the two,

$$\frac{L_v}{T} = -\frac{\partial(p, v)}{\partial(x, T)} \qquad (13)$$

which is the generalized Clapeyron-Clausius equation. Making use of this in (11) we obtain the "well-known equation" mentioned by Professor Lewis, and given by van der Waals, p. 52:

$$\frac{\partial C_v}{\partial v} = T\frac{\partial^2 p}{\partial T^2}$$

Applying this to the case mentioned by Professor Lewis,

$$p = \varphi_1(v) + T\varphi_2(v)$$

which is the case under which van der Waals' familiar equation comes, we find, to be sure, that C_v is independent of the *volume*, but on solving for T, and writing

$$T = pV_1(v) + V_2(v)$$

and applying equations (1) and (2), we do not find anything particularly simple for C_p.

All this is very simple, but what I wish to call attention to is that in any case, the specific heats being defined by a partial differential equation of the first order, there is one perfectly arbitrary function, by no means necessarily of the temperature. In any case I believe that the teaching of thermodynamics by means of the equation of the so-called ideal gas or by means of van der Waals' equation is by no means conducive to clearness.

ANOMALOUS OSMOSE

By F. E. Bartell

Chemical Laboratory, University of Michigan*

It has repeatedly been noted that solutions of electrolytes with different types of "semi-permeable" membranes give osmotic pressure values quite different in magnitude from the calculated maximum values derived from the gas law formulations. We may define *normal osmotic pressure values* as those which conform to the gas law formulations. We have an example of this type when a solution of cane sugar is used with a semi-permeable membrane such as copper ferrocyanide. *Anomalous osmotic pressure values* are those which do not conform to the gas law formulations; they may be greater or less than the normal values. The abnormality of the effects may be of considerable magnitude and may even result in so-called negative osmose, in which case the direction of flow of liquid is from the concentrated to the more dilute solution.

* Communicated by M. Gomberg, April 20, 1920

Anomalous osmose has been observed with membranes of inorganic materials, such as unglazed porcelain [1] and finely divided metals;[2] with membranes of plant tissue, such as parchment paper;[3] with membranes of animal tissue, such as gold beaters skin,[4] and with membranes of organic gelatinous materials, such as gelatine and collodion.[5] The writer has investigated each of these types of membranes and has noted their behavior when used with different types of solutions. Measurements have included osmotic pressure, the sign of the charge which exists between the membrane material and the solution, and the difference in potential which exists between the two solutions in contact with the faces of the membrane.

In order to account for abnormal osmotic effects, the following postulates have been made:

1. Capillary pores connect the two faces of an osmotic membrane. These pores become filled with liquid from the solutions bathing the membrane faces.

2. An electrical double layer may exist between the walls of the capillary pores and the liquid within them (i.e., the Helmholtz electrical layer). The magnitude of the electrical double layer depends upon the extent of the selective adsorption of the ions.

3. A difference of potential may exist between the two faces of the membrane. The magnitude of this value is primarily dependent upon diffusion, relative ionic migration velocities, and relative ionic adsorption.

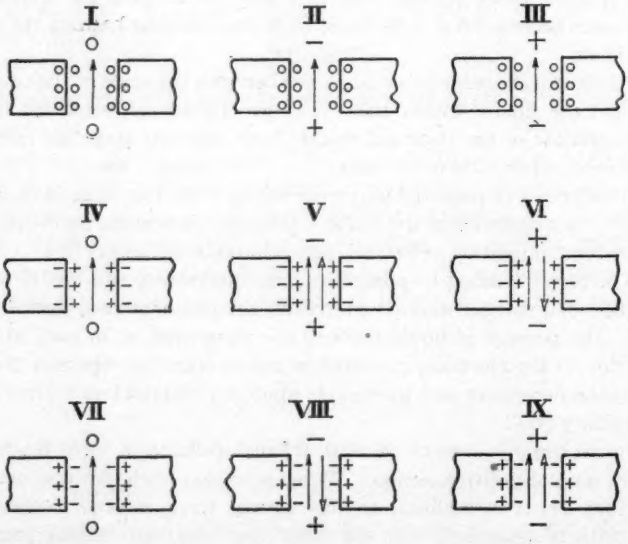
4. The forces tending to produce anomalous osmose operate in practically the same manner as do the forces in the phenomena of electrical osmose. The passage of liquid through the membrane is, in each of these cases, due to the electrical potential which is operative between the two faces of the membrane and which acts upon the charged liquid layer along the capillary wall.

5. In an osmotic system there is always operative a force tending to produce normal positive osmose. When solutions of electrolytes are used there may exist, in addition to this normal force, another force acting either with or against it, with the result that observed osmotic pressures may be either abnormally great or abnormally small.

Electrical States Which May Be Associated with the Membrane System.—According to the postulates above outlined, nine different states of electrification may exist within the osmotic system. Each of the following diagrams represents the electrical influences associated with a single pore extending through a membrane; this includes the sign of the electrostatic charge on the pore wall with the corresponding opposite charge of the liquid layer bathing this pore wall, together with the orientation of the electrical potential existing between the two faces of the membrane. The concentrated solution is in every case represented on the upper side. In each case the arrow at the left pointing upward indicates the direction of the tendency to produce normal osmose, while the arrow on the right

indicates the direction of the superimposed effect tending to produce abnormal osmose. With electrical orientations represented in cases I, II, III, IV and VII, normal osmose would be obtained; with electrical orientations represented in cases V and IX, abnormally great positive osmose would be obtained; with electrical orientations represented in cases VI and VIII, abnormally small (or negative) osmose would result.

An indication of the orientation of the electrical charge, and the magnitude of the charge, on the capillary wall system has been obtained by reducing the material of the membrane to the form of a fine suspension and then determining the electrical sign of these particles by cataphoresis, i. e., by noting the direction of movement of the fine particles when subjected to the driving force of a difference of potential.



The difference in potential between the two solutions bathing the faces of the membrane has been measured in practically all our osmotic experiments. This value has been assumed to represent the difference in potential which exists between the two faces of the membrane and to indicate at least the direction of the driving force tending to produce abnormal osmose.

It has been noted that the value of the electromotive force observed is quite different from the value calculated by assuming it to be due to contact potential. In some cases it is even opposite in sign to such calculated values.

In the following tables, the sign preceding the potential value refers to the sign of the charge on the solution side of the membrane. The (−)

sign preceding the osmose value indicates negative osmose. Some typical results are given in the following tables.

RESULTS OBTAINED WITH PORCELAIN MEMBRANE

SOLUTION	POTENTIAL OF CONCENTRATED SOLUTION SIDE	OSMOSE	SIGN OF CAPILLARY WALL	SIGN OF LIQUID IN CAPILLARY
0.1 *M* KNO_3	−0.006	+64.5	−	+
0.1 $LiNO_3$	+0.007	−31	−	+
0.1 $Al(NO_3)_3$	+0.310	+447	+	−
0.005 *M* HCl		+27		
0.01 *M* HCl	−0.011	+15	+	−
0.2 *M* HCl	+0.013	+119	+	−
0.005 *M* NaOH		+26		
0.01 *M* NaOH	−0.022	+81	—	+
0.2 *M* NaOH	+0.016	−45	−	+

RESULTS OBTAINED WITH GOLD BEATERS SKIN

SOLUTION	POTENTIAL OF CONCENTRATED SOLUTION SIDE	OSMOSE	SIGN OF CAPILLARY WALL	SIGN OF LIQUID IN CAPILLARY
0.05*M* KCl	+0.002	+ 8.5	+	−
0.05 *M* LiCl	+0.046	+ 11	+	−
0.05 *M* $AlCl_3$	+0.067	+550	+	−
0.1 HCl	−0.09	− 2	+	−
0.5 H_2SO_4	−0.094	− 40	+	−
0.5 H_3PO_4	−0.103	− 45	+	−
0.01 NaOH	+0.040	+ 5	−	+
0.5 NaOH	+0.04	− 5.5	−	+

RESULTS OBTAINED WITH POTASSIUM SALTS WITH PORCELAIN MEMBRANES

SOLUTION	POTENTIAL OF CONCENTRATED SOLUTION SIDE	OSMOSE	SIGN OF CAPILLARY WALL	SIGN OF LIQUID IN CAPILLARY
KCNS	−0.0185	+100.5	−	+
KCl	−0.0105	+ 80	−	+
KBr	−0.0090	+ 71	−	+
KNO_3	−0.0060	+ 64.5	−	+
KI	−0.0050	+ 55	−	+

The results obtained show that the magnitude of the osmose varies with the magnitude of the potential. The magnitude of osmose will not vary directly with the magnitude of potential inasmuch as the electromotive force values must overcome the tendency toward positive osmose, and, also, the value of the double electric layer of the capillary wall system is different in every solution used.

Gold beaters skin in water is slightly electro-negative. Salts with univalent cations may cause the membrane material to become positively charged. The magnitude of the charge on the capillary walls increases with increase in valency of the cation. This explains the abnormally great osmose with $AlCl_3$ solution.

It is noted that the direction and the extent of osmose in the above experiments conform to predictions which follow from the application of the postulates given above. In those cases in which the direction of flow is not that actually indicated by the sign of the charge of the system, it will be noted that the osmose is abnormal and that the tendency is operative in the direction predicted.

The experiments with various types of membranes have been extended considerably beyond those cited. Parchment paper, gold beaters skin, collodion, and gelatine membranes have been studied. The generalizations outlined above have been found to apply very satisfactorily in the case of each of these types of membranes. The results will be published in the near future.

[1] Bartell, F. E. and Hocker, C. D., *J. Amer. Chem. Soc.*, **38**, 1916 (1036).
[2] Robinson, C. S., *J. Phys. Chem.*, **22**, 1918 (99); **22**, 1918 (153).
[3] Unpublished data.
[4] Girard, P., *Rev. Gen. Sci.*, **20**, 1909 (694); Lillie, R. S., *Amer. J. Physiol.*, **28**, 1911 (194).
[5] Loeb, J., *J. Gen. Physiol.*, **2**, 1919 (87).

A PHOTOGRAPHIC METHOD OF FINDING THE INSTANTANEOUS VELOCITY OF SOUND WAVES AT POINTS NEAR THE SOURCE

By Arthur L. Foley

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Communicated by R. A. Millikan, April 10, 1920

Sparks from the condensers L, L (Leyden jars, charged by an electric influence machine not shown in the figure) pass across the gaps G, G, the sound gap S, and the illuminating gap I. The light from the spark at I casts a shadow (caused by refraction) of the sound wave produced by the spark at S upon the photographic dry plate P, provided the spark at I occurs at the proper time. The gaps I and S are in series, the spark at S occurring first, the spark at I being retarded the proper time interval by means of an adjustable condenser K.

D is a steel disk 76 cm. in diameter mounted to rotate at high speed just in front of and near the lower edge of the dry plate P. The rim of the disk was ground to a chisel edge and very small teeth were cut in it at intervals of a cm. or so. The intervals and teeth were made irregular

to insure the identification of their shadows on the photographic plate. The rotational speed of the disk was measured by means of a calibrated Van Sicklen-Elgin tachometer. A light-tight box encloses the spark gaps I and S, the dry plate P and the disk D.

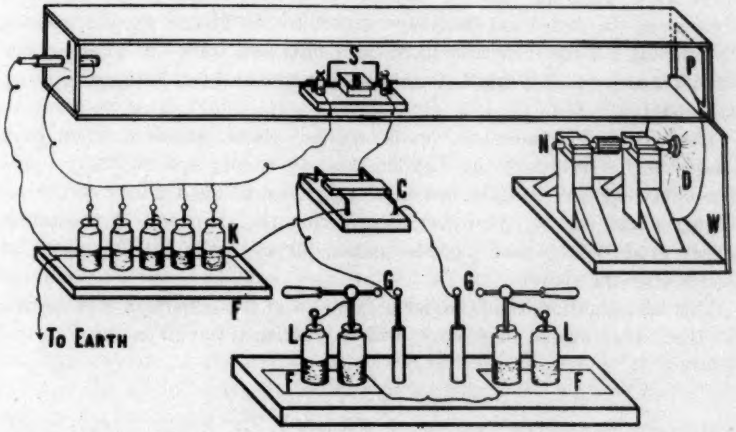


FIG. 1

When a spark occurs at S a cylindrical sound wave is generated and a shadow of the saw teeth is thrown on the photographic plate. Assuming that the time interval between S and I has been adjusted properly the spark at I casts on the dry plate a shadow of the sound wave and the teeth

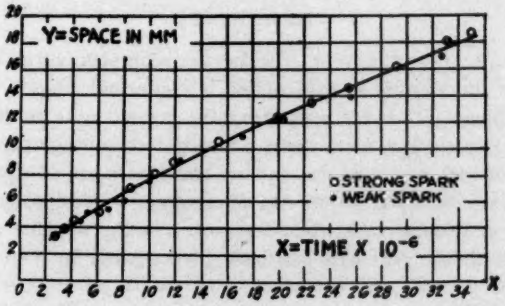


FIG. 2

of the disk; the latter having moved into a new position during the interval between the sparks. The true radius of the sound wave is calculated from the geometry of the figure and the radius as measured on the photographic plate. The time interval is calculated from the measured distance through which any given tooth has turned, together with the speed

of rotation and the distances from the plate to I, S, and the disk D. Plotting wave radii as ordinates and the corresponding time intervals as abscissas, the tangent to the curve at any point gives the instantaneous velocity of sound at that distance from the source. The accompanying curve shows that the velocity varies from about 660 meters per sec. at a distance of 3.2 mm. from the sound source to 380 meters per second at a distance of 1.8 cm. Results have been obtained since the above curve was plotted for points much closer to the source and for distances up to a half meter.

The writer designates as "weak sparks" those produced when each condenser, L, consists of two Leyden jars; as "strong sparks," seven jars. The curve appears to show but little difference in the velocity of the resulting sound waves. However, considering the shortness of the sound pulses produced by such electric sparks, all are really intense waves at points near the source.

This investigation, made under a grant from the American Association for the Advancement of Science, will be published in full in the *Physical Review*.

CONFORMAL MAPPING OF A FAMILY OF REAL CONICS ON ANOTHER

By T. H. Gronwall

Technical Staff, Office of the Chief of Ordnance, Washington, D. C.

Communicated by E. H. Moore, April 27, 1920

Note IV On Conformal Mapping Under Aid of Grant No. 207 From the Bache Fund

Let $z = x + yi$ and $w = u + vi$ be two complex variables, and let the analytic function $w = w(z)$ define a conformal map of the z-plane upon the w-plane. It is the purpose of the present note to determine all functions $w(z)$ such that there exists a family (containing at least one real parameter) of real conics in the z-plane which is mapped upon a family (obviously with an equal number of parameters) of real conics in the w-plane. The particular case when the conics in the w-plane are straight lines parallel to the real axis has been investigated by Von der Mühll[1] and Meyer.[2]

It is convenient to use the isometric coördinate $z = x + yi$ and $\bar{z} = x - yi$, and we begin by establishing the following general results:

When $\bar{z} = f(z, t)$, where f is analytic in both its arguments and t is a real parameter, represents a family of real curves in the z-plane, and similarly $\bar{w} = F(w, t)$ is a family of real curves in the w-plane, the necessary and sufficient condition that $w = w(z)$ shall map these two families upon each other is that the relation

$$\begin{vmatrix} \dfrac{\partial F}{\partial w} & \dfrac{dw}{dz} \\ \dfrac{\partial F}{\partial t} & \end{vmatrix} = \begin{vmatrix} \dfrac{\partial f}{\partial z} \\ \dfrac{\partial f}{\partial t} \end{vmatrix}$$

shall become an identity in z and t upon making $w = w(z)$.

In order that the z- and w-planes may be mapped upon an auxiliary Z-plane in such a manner that our two families of curves are mapped into the parallels to the real Z-axis, it is necessary and sufficient that the right-hand member in the relation above equals the product of a function of z by a function of t. When this is not the case, and our two families of curves are both algebraic, then $w(z)$ is necessarily an algebraic function of z.

Applying these general propositions to the various kinds of real conics, we obtain all the conformal maps having the required property, and replacing z and w by $az + b$ and $\alpha w + \beta$, respectively, or by $(az + b)/(cz + d)$ and $(\alpha z + \beta)/(\gamma z + \delta)$ when the conics involved are circles, and interchanging z and w when necessary, we may reduce our conformal maps to the following types, arranged according to the number of parameters in the families of conics:

Five parameters (one type):

$$w = z; \tag{I}$$

any conic in the z-plane is transformed into the same conic in the w-plane.

Three parameters (two types):

$$w = 1/z; \tag{II}$$

any circle or straight line is transformed into a circle or straight line.

$$w = z^2; \tag{III}$$

denoting the parameters by k, r and θ, any ellipse ($k>1$) or hyperbola ($1>k>-1$)

$$\bar{z} = k\frac{\bar{z}_1}{z_1}z + \frac{\bar{z}_1}{z_1}\sqrt{(k^2-1)(z^2-z_1^2)}$$

with center at the origin and foci at

$$z_1 = re^{\theta i},\ z_2 = -z_1$$

is transformed into the ellipse ($k>1$) or hyperbola ($1>k>-1$)

$$\bar{w} = K\frac{\bar{w}_1}{w_1}w + \frac{1-K}{z}\bar{w}_1 + \frac{\bar{w}_1}{w_1}\sqrt{(K^2-1)w(w-w_1)}$$

with $K = 2k^2-1$ and foci at

$$w_1 = z_1^2 = r^2e^{2\theta i},\ w_2 = 0.$$

In the special case $k = 0$, the equilateral hyperbolas

$$e^{-2\theta i}z^2 + e^{2\theta i}\bar{z}^2 = r^2$$

with center at the origin go into the straight lines

$$e^{-2\theta i}w + e^{2\theta i}\bar{w} = r^2$$

Two parameters, denoted by r, θ or p, θ (four types):

$$w^2 = z^2 - 1; \qquad \text{(IV)}$$

any equilateral hyperbola

$$e^{-2\theta i}z^2 + e^{2\theta i}\bar{z}^2 = r^2$$

with center at the origin and foci at $z_1 = re^{\theta i}$, $z_2 = -z_1$ is transformed into the equilateral hyperbola

$$e^{-2\theta i}w^2 + e^{2\theta i}\bar{w}^2 = r^2 - 2\cos 2\theta$$

with center at the origin and foci at

$$w_1 = \sqrt{r^2 - 2\cos 2\theta}\, e^{\theta i},\ w_2 = -w_1.$$

In the special case $r^2 = 2\cos 2\theta$, we have a pair of perpendicular straight lines through the origin in the w-plane.

$$w = z^2; \qquad \text{(V)}$$

any straight line

$$e^{1/2\theta i}\bar{z} - e^{-1/2\theta i}z = i\sqrt{\bar{p}}$$

is transformed into the parabola

$$e^{1/2\theta i}\sqrt{\bar{w}} - e\sqrt{}^{1/2\theta i}w = i\sqrt{\bar{p}}$$

with focus at the origin and at a distance of $^1/_2p$ from the directrix, θ being the angle between the axis of the parabola and the real axis in the w-plane.

$$w = (z^2 - c_1)^2, \text{ where } c_1 = 0 \text{ or } 1; \qquad \text{(VI)}$$

any equilateral hyperbola

$$e^{1/2\theta i}\bar{z}^2 - e^{-1/2\theta i}z^2 = i(\sqrt{\bar{p}} + 2c_1 \sin {}^1/_2\theta)$$

with center at the origin and foci at

$$z_1 = e^{1/4(\theta-\pi)i}(\sqrt{\bar{p}} + 2c_1 \sin {}^1/_2\theta)^{1/2},\ z_2 = -z_1$$

is transformed into the parabola

$$e^{1/2\theta i}\sqrt{\bar{w}} - e^{-1/2\theta i}\sqrt{\bar{w}} = i\sqrt{\bar{p}}$$

with focus at the origin.

$$w = (\sqrt{\bar{z}} - 1)^2; \qquad \text{(VII)}$$

any parabola

$$e^{1/2\theta i}\sqrt{\bar{z}} - e^{-1/2\theta i}\sqrt{\bar{z}} = i(\sqrt{\bar{p}} + 2\sin {}^1/_2\theta)$$

with focus at the origin is transformed into the parabola

$$e^{1/2\theta i}\sqrt{\bar{w}} - e^{-1/2\theta i}\sqrt{\bar{w}} = i\sqrt{\bar{p}}$$

with focus at the origin.

One parameter, denoted by r, θ or p (nine principal types, the linear composition of each of these with itself or with another giving a total of forty-five types). There are tabulated below the nine functions which map the one parameter families of conics specified on the parallels to the

real axis in the Z-plane. When $\varphi_1(z)$ and $\varphi_2(z)$ are any two of these functions (identical or different), all maps transforming one parameter families of conics into others are found by making

$$\varphi_2(w) = A\varphi_1(z) + B,$$

where A and B are constants and A real, and then making the linear transformations on z and w indicated at the beginning of this summary.

$$Z = z; \qquad \text{(VIII)}$$

all parallels to the real axis (special case of I).

$$Z = \log z; \qquad \text{(IX)}$$

all straight lines through $z = 0$ (the circles with center at $z = 0$ go into parallels to the imaginary Z-axis).

$$Z = i \log z; \qquad \text{(X)}$$

all circles with center at $z = 0$ (the straight lines through $z = 0$ go into parallels to the imaginary Z-axis).

$$Z = z^2; \qquad \text{(XI)}$$

all equilateral hyperbolas $z^2 + \bar{z}^2 = r^2$ with center at the origin and foci on the real axis (special case of III).

$$Z = \log (z^2 - 1); \qquad \text{(XII)}$$

all equilateral hyperbolas

$$e^{-2\theta i} z^2 + e^{2\theta i} \bar{z}^2 = 2 \cos 2\theta$$

with center at the origin and foci $\pm \sqrt{2 \cos 2\theta}\, e^{\theta i}$ (which are the end-points of the diameter with angle of slope θ of the lemniscate with foci at $+1$ and -1). These hyperbolas all pass through the foci of the lemniscate.

$$Z = \log (z + \sqrt{z^2 - 1}) \text{ or } z = {}^1/_2(e^Z + e^{-Z}); \qquad \text{(XIII)}$$

all hyperbolas with foci at $+1$ and -1 (all ellipses with these foci go into parallels to the imaginary Z-axis).

$$Z = 1/i \log (z + \sqrt{z^2 - 1}) \text{ or } z = \cos Z; \qquad \text{(XIV)}$$

all ellipses with foci at $+1$ and -1 (all hyperbolas with these foci go into the parallels to the imaginary Z-axis).

$$Z = \sqrt{z}; \qquad \text{(XV)}$$

all parabolas $\sqrt{z} - \sqrt{\bar{z}} = i\sqrt{p}$ with focus at the origin and the real axis as axis (special case of V).

$$Z = \log (\sqrt{z} - 1); \qquad \text{(XVI)}$$

all parabolas

$$e^{{}^1/_2\theta i}\sqrt{z} - e^{-{}^1/_2\theta i}\sqrt{\bar{z}} = 2i \sin {}^1/_2\theta$$

with focus at the origin and passing through the point $z = 1$.

[1] Von der Mühll, *J. Math., Berlin*, **69**, 1868 (264).

[2] Meyer, "Ueber die... Isothermen," *Dissertation*, Göttingen, 1879.

EXPERIMENTS ON THE VIBRATION OF AIR IN CONICAL HORNS

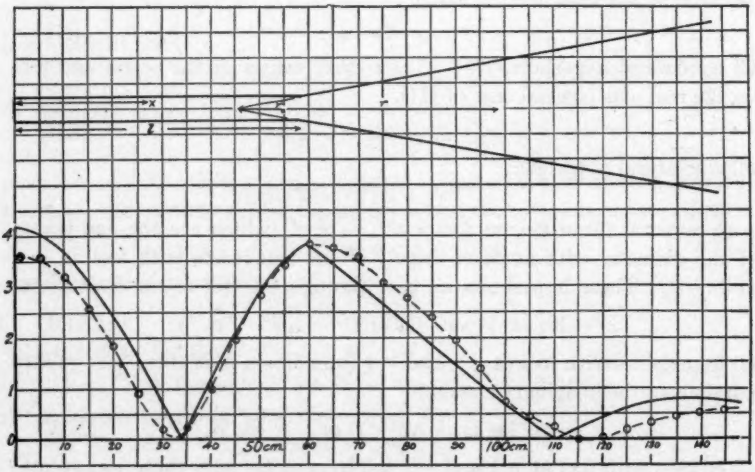
BY ARTHUR GORDON WEBSTER

CLARK UNIVERSITY, WORCESTER, MASS.

Communicated March 30, 1920

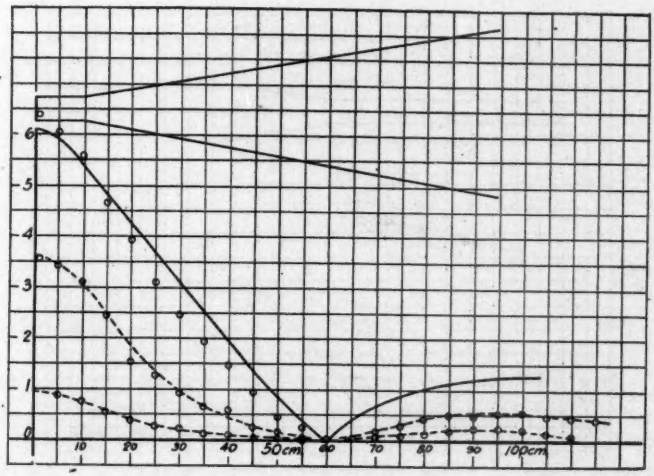
At the meeting of the Committee on Sound of the National Research Council recently held at Geneva, Illinois, it was represented to me that my habit of not publishing results had been a serious detriment to the progress of certain investigators in sound and I was strenuously urged to reform my habits. I shall, therefore, take the liberty of bringing out of my drawers a certain number of papers some of which I have had for many years but which for one reason or another have not been published.

The curves presented herewith were shown at a meeting of the American Association for the Advancement of Science in Columbus in 1915,



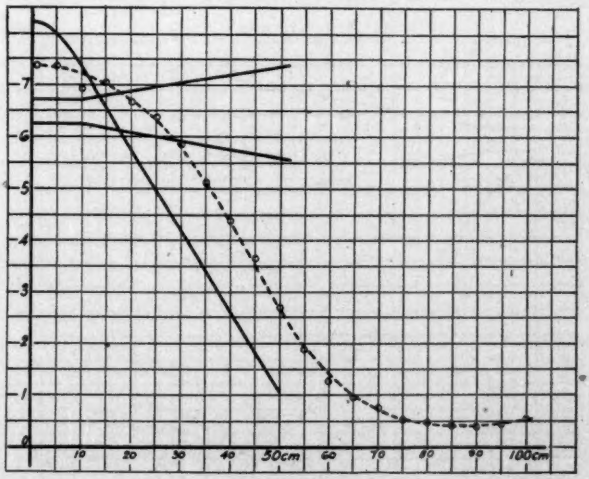
but I believe have not been published. They were made by means of the phonometer and phone described in my paper in these PROCEEDINGS, **5**, May, 1919 (163–166), and they are intended to verify the theory of horns given by me in these PROCEEDINGS, **5**, July, 1919 (275–283).

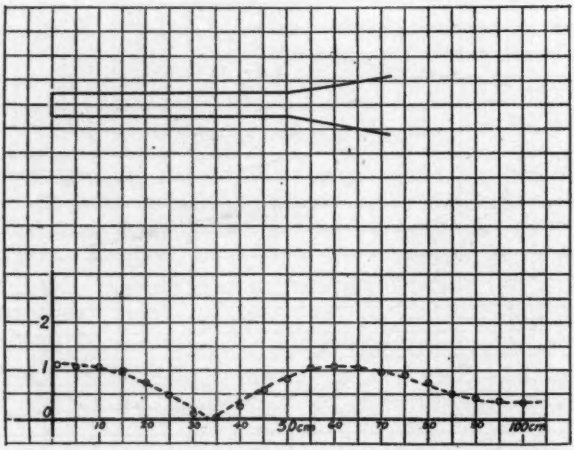
The horn is always a portion of a circular cone mounted on one end of a cylindrical tube, the other end of which is closed. The sound was made by the phone emitting a constant sound of a pitch of 256 per second. The standing waves inside the horn were explored by an antenna consisting of a glass tube of three millimeters internal diameter and several feet long attached to a disk closing the end of a cylindrical tube screwed into the opening of the phonometer. This could be put into any part of



the space to be explored. To be sure, the pressure measured by the phonometer is not the same as that of the antenna, but inasmuch as standing waves are formed in the antenna the pressure measured by the phonometer is proportional to that at the end of the antenna and, therefore, to that in the place to be observed.

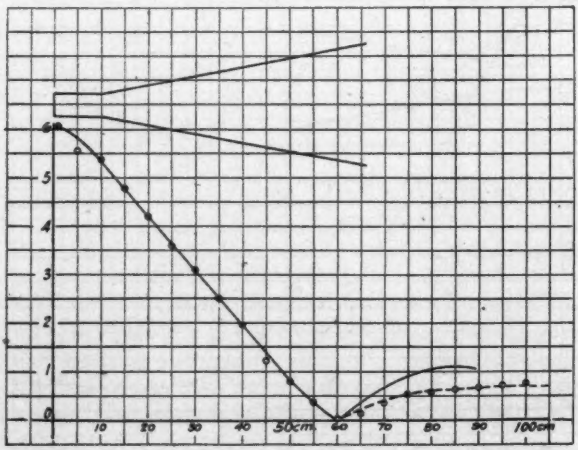
The figures printed herewith show on the dotted lines the actual observations and on the full lines the theory. The shape and dimensions of the

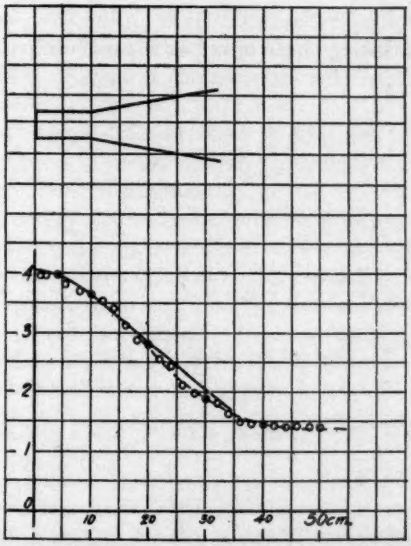




horns are also shown on the figures. The experiments were made by my then assistant, Dr. H. F. Stimson, to whom I am indebted for his careful work.

It will be seen that the theory is very substantially verified, the defect in it being due to the very rough estimate made about the correction of the open end which, as I stated in my paper on horns, is far from being accurate. But inasmuch as it is the only theory that has ever been given I feel that the results constitute a distinct advance. Prof. G. E. Stewart has done a large amount of work on horns and he also confirms my theory

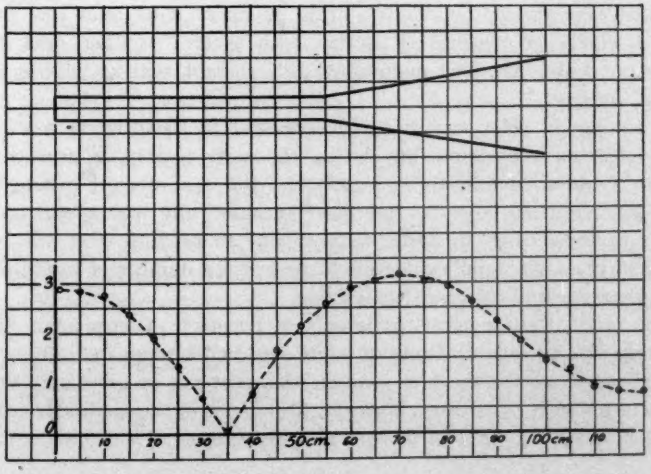




with about the same degree of accuracy as it was confirmed by these results more than four years ago. Professor Stewart has also given a theoretical treatment that is an improvement on mine. We shall see whether it agrees better with the observations. The formulae I have used in calculating these curves are:

In cylinder $$p = \frac{P \sin \beta \cos kx}{r_0 \cos kl},$$

In cone $$p = P \sin k(r - r_0 + \beta)/r,$$



$P = p_o r_o/\sin\beta$, $\tan k\beta = 1/(\operatorname{ctn} k\alpha - \tan kl)$, $\tan k\alpha = kr_o$.

I enclose the following errata in my article on horns cited above:

PAGE	EQUATION	FOR	READ
276	4	$\frac{k}{2V}i$	$\frac{k}{2\pi}i$
277	13	uv	uv
278	17	uv, αv	uv, αv
278	19–20	d	∂
281	above 36	$Z_1 = ck^2\{$	$Z_1 = ek^2\{$
281	after 37	$H.K.$	$H.F.$
282		$\frac{d^2p}{vn^2} + m\frac{dp}{vn} + p = 0,$	$\frac{\partial^2 p}{\partial x^2} - m\frac{\partial p}{\partial x} + k^2 a = 0$
		$p = e^{-\sqrt{4-k^2}x}\lvert A\cos kx + B\sin kx\rvert$	$p = e^{\frac{mx}{2}}\{A\cos k\sqrt{1-(m/2k)^2}.x + B\sin k\sqrt{1-(m/2k)^2}.x\}$
		$x = e^{-\sqrt{4-k^2}t}\lvert C\cos kn + Dmk^2\rvert$	$X = e^{\frac{mx}{2}}\{C\cos k\sqrt{1-(m/2k)^2}.x + D\sin k\sqrt{1-(m/2k)^2}.x\}$

THE RELATIVE IMPORTANCE OF HEREDITY AND ENVIRONMENT IN DETERMINING THE PIEBALD PATTERN OF GUINEA-PIGS

By Sewall Wright

Bureau of Animal Industry, United States Department of Agriculture

Communicated by R. Pearl, March 17, 1920

The Bureau of Animal Industry has been carrying on an experiment on the effects of inbreeding on guinea-pigs since 1906. Twenty-three families were started successfully from as many pairs and were maintained wholly by matings of brother with sister. Another stock from the same source has been maintained as a control, without mating even second cousins.

A number of color variations were present in the original stock and most of them kept appearing during the early history of each family as the result of Mendelian segregation. After a number of generations, however, a certain color became fixed automatically within each inbred line. In some cases an entire family came to produce only one color. In most cases, the families became broken into a number of sub-families, each characterized by a particular color.

The original stock consisted largely of tricolors. Conspicuous differences in the average amounts of white and yellow appeared among the inbred families from the first, proving the existence of hereditary differences in pattern. Contrary, however, to the case of the qualitative differences in color, no types of pattern ever became at all well fixed. Varia-

tions covering almost the entire range from solid color to solid white can be found in each line. Even after twenty generations of inbreeding, a guinea-pig with 20% of white in the coat may have a litter mate with as much as 90% of white.

As a supplement to more direct attempts to determine the cause of these variations in the pattern, it seemed desirable to find the relative importance of heredity and environmental factors.

The pattern of tricolors is really a combination of two patterns which are, in the main, inherited independently of each other, although obviously associated in development. These are the piebald pattern, consisting of colored spots on a white ground, and the tortoiseshell pattern consisting of black spots on a yellow ground. In a tricolor, a certain pattern of color and white is determined irrespective of the quality of the color. Another group of factors determines which spots shall be black (or one of the modifications of black, such as agouti, sepia or brown) and which shall be yellow or red.

A cross between a homozygous solid black and a tricolor of one of the inbred families produces black young with no yellow spots and only a very small amount of white, if any. On back-crossing with the tricolor stock, four kinds of young are produced in approximately equal numbers, viz., solid black, black and yellow tortoiseshells, black and white piebalds, and black, yellow and white tricolors. As in the first generation, there is apt to be a small amount of white in the first two classes. The nearly solid black type has been back-crossed for seven generations with pure tricolor stock, producing young with as much as 255/256 of the blood of the latter stock, with the same results as in the first back-cross, four classes of young in equal numbers. It is clear that two independent Mendelian factors are involved. One of the factors is completely dominant over that for tortoiseshell, the other is nearly but not quite fully dominant over that for piebald. In the stocks considered in the present paper both of these dominant factors were absent, leaving all of the animals tricolor except for the occasional somatic bicolors and black-eyed whites to be expected in a stock with a high average amount of white in the coat. The hereditary factors to be considered here, those by which one tricolor differs from another, are of the kind frequently known as modifiers. Crosses between inbred families at opposite extremes in amount of white have resulted in offspring which are almost exactly intermediate on the average. In the second generation, there is only a little more variability than in the first; much less than would be present if there were segregation of a single factor.

The minor as well as the major variations of the piebald and tortoiseshell patterns appear to be largely independent of each other, although at least one factor has been discovered which affects both, viz., sex. The males of all stocks have on the average some 7% more color than the females

and also some 4% more black within the colored spots. In the present paper only variations in the piebald pattern will be considered and the sexes will be dealt with separately.

A record of the patterns of the guinea-pigs has been kept from the first by drawing the spots within a rubber-stamp outline. Only as much of the pattern as is visible from the back was recorded in this way before 1915. Since that date, the patterns of both the back and belly have been recorded, but separately, making comparison with the earlier records possible. The amount of white in each case has been estimated by means of a piece of tracing cloth on which the area in the stamped outline is divided into ten equal dorsal parts and ten equal ventral parts, including the legs. Each guinea-pig is given a record such as B-R-W 2–5–3/X–5–5, meaning 20% black, 50% red and 30% white on the back, a trace of black, 50% red and 50% white on the belly. In the present paper only the dorsal areas are considered. As the correlation between the amount of white on the back and belly is 0.86 and the back contains the most persistent center of color (the ear region) and almost the most persistent white (the nose) the grades, based on the dorsal part only, are believed to be practically as satisfactory as grades based on the entire coat.

Figure I shows the distribution of grades of white in three of the inbred families and in the control stock in 1916 to 1918. The representatives of

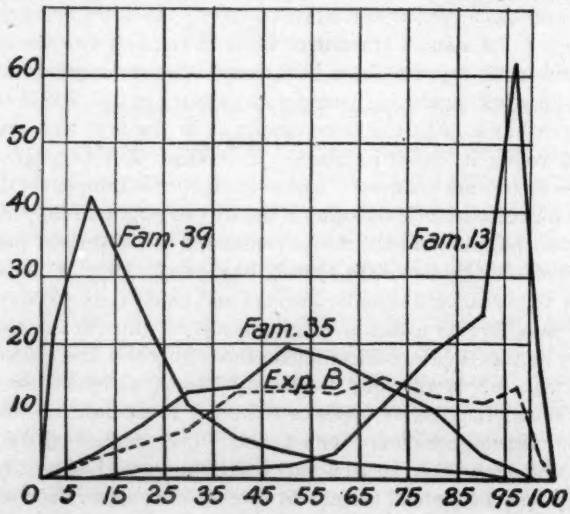


FIG. 1

Percentage distribution of grades of white in three inbred families (39, 35, 13) and in the control stock (B) during 1916–1918. The percentages with less than 5 per cent white and with 95 to 100 per cent white are given double value, the class ranges being of half size. The few solid whites in Family 13 and B are arbitrarily given double value.

family 39 during this period were all descended from a single mating in the fourth generation of inbreeding, and with a few exceptions, from one in the eighth. Family 35 was descended wholly from one mating in the seventh generation and largely from a single one in the twelfth. Family 13 was descended wholly from one mating in the second generation and largely from one in the seventh.

It will be seen that each family varies almost through the entire range, but that the mean values are markedly different. Family 39 has only about 20% white on the average, family 13 at the opposite extreme has about 85%, while family 35 has its average near 50%. On attempting to

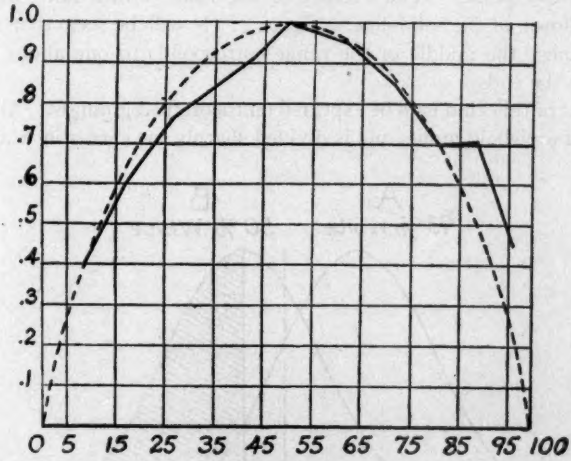


FIG. 2

The solid line shows the average distance between quartile and median in distribution curves from different inbred families in which the midpoint between quartile and median falls at different percentages of white. Based on 68 quartiles. The average quartile distance near 50 per cent white is taken as the unit. The broken line shows the expected quartile distances according to the hypothesis discussed in the text.

compare the variability of the families, a difficulty at once becomes apparent. The distribution curve of family 35 is practically symmetrical, but those of families 39 and 13 are markedly skew in opposite senses. This is not because of overstepping of the physiological limits, as there were no solid colored guinea-pigs in family 39, and only 8% black-eyed white in family 13. The curves appear to be compressed at the ends of the range, indicating that a unit of variation in these regions means more than in the middle of the range. It is especially difficult, so to speak, to remove the last bit of color, or the last bit of white. This phenomenon is shown by all of the stocks in which there is not an approximately equal average amount of color and white.

It is clear that measures of variability have little value unless the grades are properly corrected. Two methods have been tried for making this correction, one empirical, the other theoretical. The distribution of grades of white was tabulated, males and females separately, in the 17 families which were still on hand during 1916 and 1917. With a properly corrected system of grades the variability should be approximately equal in all cases. The points were found which divided each distribution into four equal parts, i.e., the median and two quartiles. The distance between the median and each quartile was then correlated with the grade midway between the two points. The average of the values which fall within one grade is shown in the solid line in figure 2. It will be seen that a given variation near the middle of the range corresponds to one about half as great near the ends.

An effect of this kind is to be expected on theoretical grounds. Although the skin of a piebald guinea-pig is divided sharply into areas in which pig-

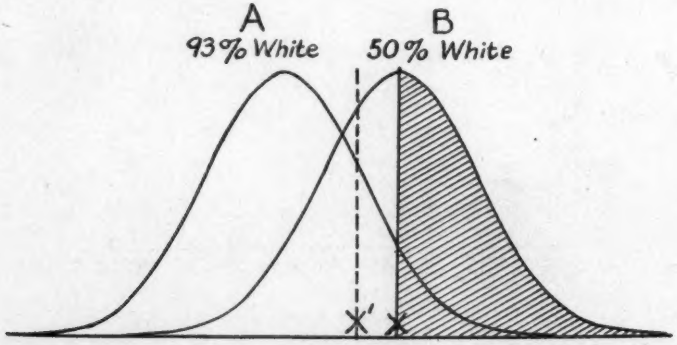


FIG. 3

The two curves are intended to represent the distribution of color potentialities among areas in the skins of two guinea-pigs. X is the critical potentiality, above which color is developed. The same change in average potentiality increases the amount of color in the guinea-pig with a large amount of white (A) only half as much as in the guinea-pig which is half white and half colored (B).

ment is either produced to the full amount characteristic of the animal, or is wholly absent, it is not to be supposed that the influences which at some critical period in ontogeny determine whether a region is to be colored or white, are so sharply alternative in themselves. It seems more reasonable to suppose that the sums of the favorable and unfavorable influences in different parts of the skin could be arranged in a graded series. Doubtless in certain white regions a slight difference in the conditions would have enabled color to develop, while in others, a great change would have been necessary. Similarly with colored areas. Suppose, then, that the skin is divided into a large number of equal areas and that it were possible

to determine the true potentiality of each area at the critical period in development. Let us assume that these potentialities are distributed, at least roughly, according to the normal probability curve.

In figure 3 variation in the piebald potentiality is measured along the horizontal axis. Curve A is intended to represent the number of areas with each potentiality in one guinea-pig, while curve B represents the same for another guinea-pig in which the tendency toward color production is higher as a whole. The critical potentiality is at X. All areas which exceed this produce color, while those which fall below, however slightly, remain white. Guinea-pig A is thus about 93% white, while B is about 50% white. It is easy to see that any cause which increases the potentiality

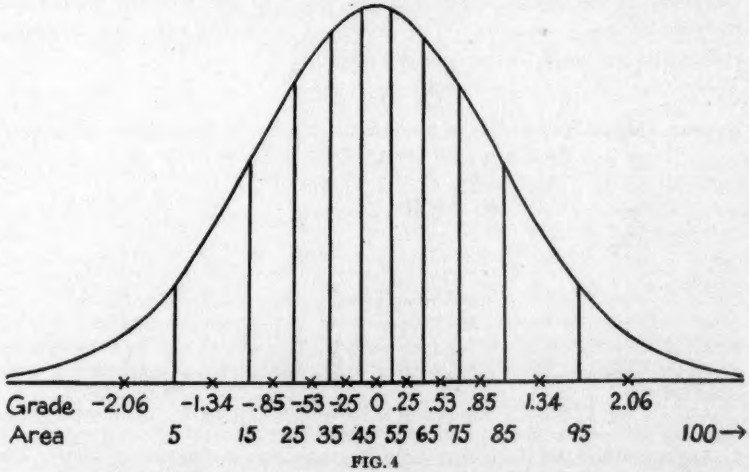


FIG. 4

A normal probability curve, the area of which is divided into tenths with the exception of the end areas which are twentieths. The deviations from the middle of the curve to the middle of each area (mean of the end areas), measured in terms of the standard deviation, are used as a corrected series of grades in place of the actual percentages of white in piebald guinea-pigs.

of A and B by a given amount makes a greater change in the area of color in B than in A. From a table of probability integrals, one can find at once the change in the area of the curve, brought about by a unit change in the value of X, in the neighborhood of any given area. These results should be comparable with those obtained empirically for physiologically equivalent variations at different areas of white.

The theoretical curve, as shown by the dotted line in figure 2, does not fit the empirical curve perfectly, but appears to give a sufficiently accurate correction of the grades for practical purposes. Aside from its greater smoothness, the theoretical curve has the advantage that by its use the variability in the amount of color in a stock of guinea-pigs can be compared

with the theoretical variability in the potentialities within a single average guinea-pig.

The relative importance of unit variations in area in different parts of the range is as the reciprocal of the variations which are physiologically equivalent. In figure 4 the area of a probability curve is divided into equal tenths except for the end areas, which are twentieths. The difference in potentiality corresponding to any difference in area can easily be estimated. The corrected series of grades as given in figure 4 has been made by finding the average of the deviations from the median of the curve to each of the class limits, with the exception that the deviation of the mean has been used for the end classes. A little consideration will show that the proper centering of the grades varies slightly with the mean of the population with which one is dealing. The grades as corrected here, are, however, sufficiently accurate for the present purpose.

TABLE 1

NUMBERS, MEANS AND STANDARD DEVIATIONS IN RANDOM-BRED STOCK AND INBRED FAMILY NO. 35 DURING 1916, 1917 AND 1918

	EXP. B (RANDOM-BRED)			FAMILY 35 (INBRED)		
	No.	Mean	σ	No.	Mean	σ
Sire.....	105	+0.337 ± 0.050	0.756 ± 0.035	73	−0.071 ± 0.044	0.559 ± 0.031
Dam.....	107	+0.390 ± 0.054	0.824 ± 0.038	73	+0.220 ± 0.045	0.570 ± 0.032
Son.....	498	+0.273 ± 0.024	0.808 ± 0.017	235	−0.017 ± 0.026	0.593 ± 0.019
Daughter	488	+0.543 ± 0.024	0.796 ± 0.017	236	+0.140 ± 0.027	0.613 ± 0.019
Total Offspring	986	+0.407 ± 0.017	0.802 ± 0.012	471	+0.062 ± 0.019	0.603 ± 0.013

Compare with figure 4 for conversion of grades into percentages of white. The standard deviation of 0.603 in the inbred family means about 22% of the total dorsal area of the coat.

Two stocks were chosen for study of the relative importance of heredity and environment. One of these was the control stock (B) in which matings were made at random, except that even the mating of second cousins was avoided. The other was family 35, which since 1915 has been descended entirely from a single mating in the seventh generation of inbreeding and largely from a single mating in the twelfth generation. The records for 1916, 1917 and 1918 were used in both cases. Table 1 gives the number in each experiment, the means and standard deviations. Table 2 gives the correlations between parent and offspring, between litter mates and between the parents, weighted by the number of young. Inspection of the tables shows that sex makes no significant difference in the correlations. This is in agreement with the results of direct experiments. Reciprocal crosses between inbred families at the opposite extremes, as families 39 and 13 in figure 1, produced practically the same result, the progeny in

both cases being almost exactly intermediate. Sex also makes no significant difference in variability. In the case of the means, however, there is in all cases the sex difference already mentioned. The mean and standard deviations are measured on a scale in which unity is the standard deviation of color potentialities for areas within the skin of a single average guinea-pig. It is interesting to find that even the random-bred guinea-pigs differ less among themselves in average color potentiality than do the spearate skin areas of a single animal.

TABLE 2

CORRELATIONS BETWEEN PARENTS, BETWEEN PARENT AND OFFSPRING AND BETWEEN LITTER MATES IN RANDOM-BRED STOCK AND INBRED FAMILY NO. 35, DURING 1916, 1917 AND 1918

	EXPERIMENT B (RANDOM-BRED)		FAMILY 35 (INBRED)	
	No.	Correlation	No.	Correlation
Sire-Dam	105	+0.019 ± 0.066	73	+0.029 ± 0.079
Sire-Son	492	+0.231 ± 0.029	235	+0.013 ± 0.044
Sire-Daughter	484	+0.194 ± 0.030	236	+0.082 ± 0.044
Dam-Son	498	+0.251 ± 0.028	235	+0.042 ± 0.044
Dam-Daughter	488	+0.165 ± 0.030	236	−0.080 ± 0.044
Average, Parent-Offspring	1962	+0.211 ± 0.015	942	+0.014 ± 0.022
Brother-Brother	390	+0.219 ± 0.033	182	+0.090 ± 0.050
Brother-Sister	437	+0.228 ± 0.031	203	+0.062 ± 0.047
Sister-Sister	406	+0.180 ± 0.032	194	+0.064 ± 0.048
Average, Litter Mates	1233	+0.214 ± 0.018	579	+0.069 ± 0.028

The matings in the correlation between sire and dam are each weighted by the number of offspring. The probable errors in this case are based merely on the number of matings. The probable errors in the other cases are based on the number of entries in the tables. Owing to repetition of individuals, they are probably somewhat too small. The correlation between litter mates in experiment B is based on 894 individuals in litters in which two or more were graded. The probable error based on this number is ±0.022 instead of ±0.018 as given in the table. Similarly there were 426 individuals in such litters in family 35, giving a probable error of ±0.031 instead of ±0.028.

The negligible correlations between the parents (+0.019 in the control stock and +0.029 in the inbred family) show that matings actually were made at random as far as color is concerned. In the control stock, both the average correlation between parent and offspring (+0.211) and that between litter mates (+0.214) are unquestionably significant. In the inbred family, there is virtually no correlation between parent and offspring (0.014) while that between litter mates is of doubtful significance (+0.069).

There is evidently very little genetic variability left in family 35, but is a good deal in the control stock. A closer analysis can be made by con-

sidering the relations between parent and offspring as shown diagrammatically in figure 5.

In a broad sense, the peculiarities of an individual are entirely determined by heredity and environment. In the present and doubtless most other cases, the latter should be divided into two elements. Nearly all tangible environmental conditions—feed, weather, health of dam, etc., are identical for litter mates. There may, indeed, be some differences in the blood supply, but in the main, differences which are not genetic must be due to irregularities in development due to the intangible sort of causes to which

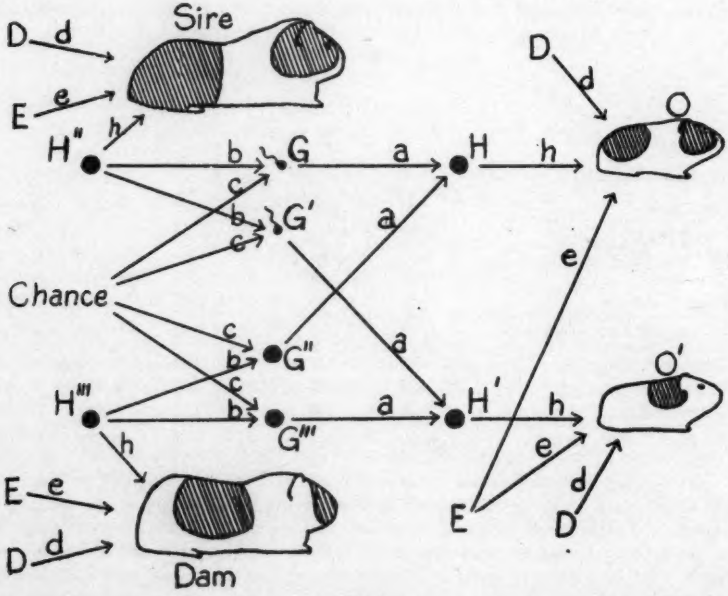


FIG. 5.

Diagram illustrating the casual relations between litter mates (O, O') and between each of them and their parents. H, H', H'', H,''' represent the genetic constitutions of the four individuals, G, G', G'', and G''' that of four germ cells. E represents such environmental factors as are common to litter mates. D represents other factors, largely ontogenetic irregularity. The small letters stand for the various path coefficients.

the word chance is applied. Variations which are due to inaccuracy in grading and are, therefore, merely apparent, cannot, however, be distinguished from those of this class. Checks have been made which indicate that apparent variability due to this cause is small in comparison with the large amount of variability which is found to be due neither to heredity nor to tangible environmental conditions.

In the diagram, the pattern of each guinea-pig is represented as determined by three factors, H (heredity), E (environment common to litter mates before birth) and D (the residue, largely irregularity in development). Our problem is to determine the degree of determination by each of these factors.

In a forthcoming paper, a method of estimating the degree to which a given effect is determined by each of a number of causes will be discussed at some length.

Figure 6 is meant to illustrate a system in which the variations of two quantities X and Y are determined in part by independent causes, such as A and D, respectively, and in part by common causes such as B and C. These common causes may be correlated with each other as in the figure. It is assumed that all of the relations are approximately linear and that the influence of the various causes are combined approximately by addi-

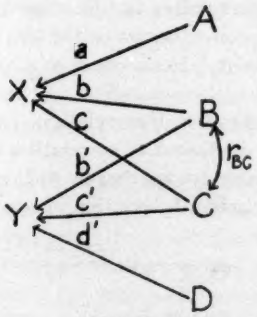


FIG. 6

Diagram illustrating two effects (XY) which are determined in part by the same correlated causes (BC).

tion. The path coefficient, measuring the importance of a given path of influence from cause to effect, is defined as the ratio of the variability of the effect to be found when all causes are constant except the one in question, the variability of which is kept unchanged, to the total variability. Variability is measured by the standard deviation. The path coefficients in the figure are represented by small letters.

It can be shown that the squares of the path coefficients measure the degree of determination by each cause. If the causes are independent of each other, the sum of the squared path coefficients is unity. If the causes are correlated, terms representing joint determination must be recognized. The complete determination of X in figure 6 by factor A and the correlated factors B and C, can be expressed by the equation:

$$a^2 + b^2 + c^2 + 2bcr_{BC} = 1 \qquad (1)$$

The squared path coefficients and the expressions for joint determination measure the portion of the mean square deviation of the effect due to the causes singly and jointly, respectively.

The correlation between two variables can be shown to equal the sum of the products of the chains of path coefficients along all of the paths by which they are connected. In figure 6

$$r_{XY} = bb' + cc' + br_{BC}c' + cr_{BC}b'. \qquad (2)$$

The path coefficients in a system of causes and effects can be calculated if a sufficient number of simultaneous equations can be made, expressing the known correlations in terms of the unknown path coefficients as in the last equation (2), and expressing complete determination of the effects by their causes as in equation (1).

The correlation between characteristics of parent and offspring are represented in figure 5 as tracing back to a single cause, the genetic constitution of the parent. Litter mates on the other hand are correlated for three reasons, the genetic constitutions of the sire and dam, respectively, and the common environment. In the absence of assortative mating, these causes are not correlated with each other. In such a system, the path coefficients are merely coefficients of correlation.

That the square of the coefficient of correlation measures the degree of determination of one variable by another in such cases may be seen from Pearson's fomula for the standard deviation of one variable (X) when another (A) is constant.

$$_A\sigma_X = \sigma_X\sqrt{1 - r^2_{XA}} \qquad (3)$$

$$r^2_{XA} = 1 - \frac{_A\sigma^2_X}{\sigma^2_X} \qquad (4)$$

The expression $1 - {}_A\sigma^2_x/\sigma^2_x$ is obviously the portion of the mean square deviation of X which is directly due to A if the latter is a cause of variation in X. In the present case the degree to which pattern is determined by the three independent factors, heredity (H), tangible environment (E) and irregularity in development (D) are measured by r^2_{OH}, r^2_{oE} and r^2_{OD}. Using the letters of figure 5 for the correlations, we have

$$h^2 + e^2 + d^2 = 1 \quad \text{from (1).} \qquad (5)$$

The correlation between parent and offspring is the product of the correlations along the single chain of factors by which they are connected.

$$r_{PO} = hbah = abh^2 \qquad (6)$$

The correlation between litter mates is the sum of the products of the correlations along the three connecting paths,

$$r_{OO'} = habbah + habbah + e\varepsilon = 2a^2b^2h^2 + e^2. \qquad (7)$$

In a sense, the genetic constitution of the fertilized egg is simply the sum of the constitutions of the egg and sperm. The latter are independent

of each other in a population which is mating at random. They are of equal importance, as indicated by the equality of inheritance from sire and dam. The degree of determination in each case must, therefore, be one-half.

$$r_{GH} = a_2 = 1/2$$
$$a = \sqrt{1/2} \qquad (8)$$

The relation between the germ-cells and the genetic constitution of one of the parents is not quite so obvious. It is well known that in a population which is mating at random, the proportion of homozygous and heterozygous classes come to an equilibrium such that the number of those heterozygous in a given factor equals twice the square root of the product of the numbers in the two homozygous classes. For factors A and a, the proportion in the three classes can be expressed in the following form:

$$x^2AA + 2xyAa + y^2aa$$

In the production of this population by the preceding generation, there were $x^2 + y^2$ cases in which similar germ-cells united at fertilization and $2xy$ cases in which the egg and sperm were dissimilar.

When this population produces germ-cells, there are $x^2 + y^2$ cases in which gametogenesis results in similar ones, and $2xy$ cases in which they are dissimilar.

Thus in a population of this kind, the correlation between the germ-cells produced by gametogenesis is exactly the same as that between two germ-cells which unite at fertilization, the correlation in which case, of course, is zero. Consequently, the relation between a germ-cell and the genetic constitution of the parent is the same as its relation to that of the offspring. Thus we may look upon the constitution of the germ-cell as half determined by that of the parent and half by chance at segregation.

$$r_{H'G} = b = a = \sqrt{1/2} \qquad (9)$$

In considering the genetic constitution of the fertilized egg as simply the sum of the constitutions of the egg and sperm, each heterozygous class is rated as if exactly intermediate between the two corresponding homozygous classes. This is not, however, the correct way to grade the genetic constitution as a factor in development, unless dominance is lacking. The correlation between the two methods of grading depends on the degree of dominance and on the ratio of dominant to recessive factors in the population. For perfect dominance and a distribution of all pairs of allelomorphs according to the proportion $1AA : 2Aa : 1aa$, the correlation is $\sqrt{2/3}$. The correlation between the genetic constitution of parent and offspring is thus about $1/2$ if there is perfect dominance, instead of $1/2$.

As already noted, the progeny of a cross between two grades of piebald are almost exactly intermediate on the average and the second generation is only slightly more variable than the first. These results might be due to two groups of dominant factors with opposed effects. The most prob-

able explanation, however, is that dominance is lacking and this view will be adopted provisionally.

With this assumption, our three equations for determining the values of h^2, e^2 and d^2 are as follows in the two stocks under consideration.

	EXP. B	FAM. 35
(5) $h^2 + e^2 + d^2 = 1$		
(10) from (6), $r_{PO} = {}^1/_2h^2 =$	0.211	0.014
(11) from (7), $r_{OO'} = {}^1/_2h^2 + e^2 =$	0.214	0.069
$h^2 =$	0.422	0.028
$e^2 =$	0.003	0.055
$d^2 =$	0.575	0.917
	1.000	1.000

Thus in the control stock, variations in pattern are determined about 42% by heredity, and 58% by irregularity in development, leaving nothing for tangible environmental factors. In the inbred family the corresponding figures are 3% for heredity, 5% for tangible environment and 92% for irregularity in development. We can find the mean squared deviation due to each kind of factor by applying these percentages to the total squared standard deviation of each stock. (Equation 3 and table 1.)

	CONTROL (B)	INBRED (35)
Heredity, $\sigma^2_{O.H}$	0.271	0.010
Tangible environment, $\sigma^2_{O.E}$	0.002	0.020
Development, $\sigma^2_{O.D}$	0.370	0.334
Total, σ^2_O	0.643	0.364

If these results had been obtained in the control stock before the inbreeding commenced, the extent to which the variability of the stock should be reduced by inbreeding could have been predicted, assuming dominance to be lacking. Inbreeding automatically renders all Mendelian factors homozygous and, therefore, eliminates all genetic variability. There is no reason for supposing that variability due to environmental or ontogenetic causes would be affected. The sum of the mean square deviations due to the latter causes is 0.372 in the control stock. This, then, is the mean square deviation to be expected after prolonged inbreeding, while its square root, 0.610, is the expected standard deviation. The actual figures in the inbred family, 0.364 for the mean square deviation, 0.603 for the standard deviation, are an even better check than could reasonably be expected on the accuracy of the assumptions on which the degrees of determination were calculated.

FOSSIL PLANTS FROM THE LATE CRETACEOUS OF TENNESSEE

BY EDWARD W. BERRY

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Communicated by Harry Fielding Reid, April 14, 1920

That part of the Atlantic Coastal Plain known as the Mississippi Embayment is one of strategic paleobotanical interest. It is a region unacquainted with orogenesis and remote from vulcanism, a region on the threshold as it were of the present-day tropics, far removed from glaciation, and with a relatively simple geological history. Hence the Gulf Coastal Plain affords a unique opportunity for studying the almost unbroken evolution of floras from the Cretaceous to the present. Many of these floras have been described in recent years, the two large unknown gaps in the record having been a knowledge of the later Upper Cretaceous and the Miocene floras. The latter still remains to be discovered, but information regarding the former will now be available through the fortunate discoveries by Dr. Bruce Wade during the summer of 1919. In working near the boundary between the Coastal Plain and the Paleozoic rocks which form its margin and basement in western Tennessee several clay lenses with an abundant varied and well-preserved representation of the flora of the Ripley formation were discovered. The Ripley formation is the latest Cretaceous known in this region and is overlain unconformably by the basal Eocene. Its sediments, in general shallow water marine sands, with many suggestions of plants in their carbonaceous sdum have heretofore yielded but few and fragmentary representatives of the forests which clothed the shores of the Ripley sea—such things as leaves or broken twigs of Araucarian or other conifers, and a few of the more coriaceous dicotyledonous leaves that chanced to settle in its waters. The clay lenses in Henry and Carroll counties are of fine-grained and pure material, evidently the sediments of quiet estuarine or lagoonal waters close to a forested shore and perhaps the distributaries of slow streams or bayous traversing a richly forested country. The collected representatives of this flora contain some surprises in the survival of what have been considered earlier forms, and have an important bearing on correlation as well as affording a fair picture of the general facies. These considerations warrant a preliminary announcement.

The most extensive best-preserved and earliest studied Upper Cretaceous floras, both in this country and abroad, have been those in the continental deposits immediately preceding, or in the initial deposits of the transgressing Upper Cretaceous sea, such for example as the Dakota sandstone of Kansas and Nebraska, the Raritan clays of New Jersey, the Perucer beds of Bohemia or the Niederschoena deposits of Saxony. Coming up-

ward in all of the sections the plant evidence has rapidly diminished in both variety and quality. A knowledge of the Ripley flora not only serves to offset this condition and furnish a basis for more precisely evaluating the passing of Cretaceous time, but it introduces us to many of the smaller and more delicate forms that almost invariably fail of preservation in coarse deposits.

Examples of the relative paucity of the later Cretaceous floras of the Atlantic Coastal Plain are furnished by noting that the Tuscaloosa, Raritan and Magothy floras comprise about 150 species each whereas when I published the Upper Cretaceous Floras of the World in 1916 I listed but 9 species from the Ripley formation in Georgia, two from beds of that age in Alabama and three from Tennessee. A foreign parallel may be taken from the Bohemia area. Here the Perucer beds (Cenomanian) have furnished 184 species, the Turonian 47 and the Chlomeker beds (Emscherian) but 32.

The present discoveries disclose the remains of 124 species complete enough for descriptive purposes, of which 86 are new to science. They are referred to 62 genera in 38 families and 25 orders. Thirty of the genera or 49% are extinct, and all but one of the species and about a dozen of the genera are not found in the Eocene.

Contrasts with the earlier Upper Cretaceous floras are the scanty representation of the gymnosperms, here reduced to 6 species, and the absence of traces Sequoia or cycads. Familiar types not present are Magnolia, and Diospyros, and there are no certainly identified Proteaceae. In a general way the flora suggests slightly warmer climates than those that preceded it during Upper Cretaceous times, and there are a considerable number of types that appear to be ancestral to members of the lower Eocene flora of the Mississippi Embayment region.

The flora contains but 5 ferns, 7 monocotyledons and 105 dicotyledons, of which last 94 are Choripetalae and 11 Gamopetalae. The largest alliances are the Leguminosae with 14 species, the Lauraceae with 12, the Myricaceae with 11 and the Moraceae with 10. The largest genera are Myrica with 11 species, Ficus with 9 and Celastrophyllum with 6.

Compared with floras in other areas the Ripley flora of Western Tennessee shows little in common with the Laramie flora of the West. It does contain a number of elements common to the Montana Group floras, particularly the flora of the Vermejo formation of Colorado and New Mexico. There are also similarities with the flora of the Patoot beds of Greenland and the upper Emscherian and lower Aturian (Campanian) floras of Western Europe.

A complete account of the Ripley flora has been prepared which will be published by the U. S. Geological Survey.

NATIONAL RESEARCH COUNCIL

NOTES FROM THE MINUTES OF THE MEETINGS IN JANUARY AND FEBRUARY, 1920, OF THE EXECUTIVE BOARD AND INTERIM COMMITTEE OF THE NATIONAL RESEARCH COUNCIL.

At the meeting of the Executive Board, February 14, the Chairman of the Committee on Publication reported that the first number of the Bulletin Series had been issued and that five numbers of the Reprint and Circular Series are in the hands of the printers. A considerable number of subscriptions have already been received for the Bulletin Series and requests from several libraries and individuals have been received for the Reprint and Circular Series as the numbers become available.

The Chairman of the Research Information Service reported the status of several projects which are being undertaken by that Division; first, the preparation of a catalogue of research laboratories or organizations to include all laboratories throughout the country in which research is fostered; second, the preparation of information concerning current investigations which will involve securing data from five or six hundred educational institutions and from thousands of independent workers and laboratories; and, third, a file of the scientific personnel of the United States, for which the names of about twelve thousand investigators have already been collected.

The Chairman of the Division of Physical Sciences presented a resolution passed by the Mathematical Association of America requesting representation in the Council through a separate Division of Mathematics. The matter was discussed at some length and referred to the Division of Physical Sciences for recommendation.

The Chairman of the Division of Chemistry and Chemical Technology stated that the annual dues of the United States to the International Chemical Union are 4500 francs, and that, at the request of the constituent societies of the Division that they be permitted to share in this expense, an agreement has been reached whereby the Division of Foreign Relations shall pay one-half the amount, the balance to be paid by the Division of Chemistry and Chemical Technology through its constituent societies.

The American Geophysical Union was made a committee of the Executive Board of the National Research Council.

LECTURE AND EXHIBIT

Mr. J. J. Carty, Vice-President of the American Telephone and Telegraph Company and member of the Executive Board and of the Division of Research Extension of the National Research Council, gave a lecture, under the auspices of the Council, February 6, in the auditorium of the new National Museum on "Science and the Industries." This lecture was illustrated by an exhibition of a number of pieces of apparatus of great

historical interest, among them being the electromagnet and galvanometer used by Joseph Henry and the original telephone used by Alexander Graham Bell and Thomas A. Watson. The lecture was also illustrated by talking motion pictures.

On February 9, an exhibit of apparatus and motion pictures telling the story of the historical development of wireless telegraphy and telephony was opened to the public in the exhibit room of the Council in Washington. This exhibit was prepared and installed by the American Telephone and Telegraph Company and the Western Electric Company, Incorporated, in coöperation with the U. S. Signal Corps and Air Service.

DIVISION AND COMMITTEE APPOINTMENTS

Government Division.—Vice-Chairman, Carl L. Alsberg, Chief, Bureau of Chemistry, Department of Agriculture; Executive Committee, Chairman, Charles D. Walcott, Secretary, Smithsonian Institution, Vice-Chairman, Carl L. Alsberg, Secretary, R. M. Yerkes, National Research Council, Col. Colden L'H. Ruggles, Ordnance Department, U. S. A., Rear Admiral Ralph Earle, Bureau of Ordnance, U. S. N., S. W. Stratton, Chief, U. S Bureau of Standards, and George Otis Smith, U. S. Geological Survey.

Representatives of the Departments of the Government.—Department of State: Julius G. Lay, Acting Foreign Trade Advisor.

Department of Treasury: Assistant Surgeon General J. W. Schereschewsky, Public Health Service.

Department of War: Maj. General William L. Sibert, Chemical Warfare Service, Maj. General George O. Squier, Signal Corps, Brig. General Marlborough Churchill, Military Intelligence Division, Col. Colden L'H. Ruggles, Ordnance Department, Col. Frederick F. Russell, Medical Corps, Col. Clarence O. Sherrill, Corps of Engineers, Lieut. Col. Robert E. Callan, Coast Artillery, Lieut. Col. Byron Q. Jones, Air Service, Major William M. Fassett, General Staff.

Department of Justice: John T. Creighton, Bureau of Investigation.

Post Office Department: John T. Miller, Chief of the Division of Correspondence.

Department of the Navy: Rear Admiral A. P. Niblack, Office of Naval Intelligence of Office of Naval Operations, Rear Admiral J. A. Hoogewerff, Bureau of Navigation, Rear Admiral D. W. Taylor, Bureau of Construction and Repair, Rear Admiral R. S. Griffin, Bureau of Steam Engineering, Rear Admiral W. C. Braisted, Medical Corps, Bureau of Medicine and Surgery, Rear Admiral C. W. Parks, Bureau of Yards and Docks.

Department of the Interior: M. H. Coulston, Assistant Commissioner of Patents, S. P. Capen, Specialist in Higher Education, Bureau of Education, George Otis Smith, Director, U. S. Geological Survey, A. P. Davis,

Director and Chief Engineer, U. S. Reclamation Service, F. G. Cottrell, Assistant Director, Bureau of Mines.

Department of Agriculture: C. F. Marvin, Chief of Weather Bureau, J. R. Mohler, Bureau of Animal Industry, K. F. Kellerman, Bureau of Plant Industry, H. S. Graves, Forest Service, C. L. Alsberg, Bureau of Chemistry, Milton Whitney, Bureau of Soils, L. O. Howard, Bureau of Entomology, E. W. Nelson, Bureau of Biological Survey, T. H. MacDonald, Bureau of Public Roads.

Department of Commerce: S. W. Stratton, Director, Bureau of Standards, Hugh M. Smith, Commissioner of Fisheries, Joseph A. Hill, Chief Statistician, Division of Revision and Results, Bureau of Census, William Bowie, Hydrographic and Geodetic Engineer, Coast and Geodetic Survey.

Department of Labor: Royal Meeker, Commissioner of Labor Statistics.

Smithsonian Institution: Charles D. Walcott, Secretary.

Division of States Relations.—Dr. T. S. Palmer, Expert in Game Conservation, Bureau of Biological Survey, as member at large.

Division of Research Extension.—An advisory Committee on Alloys Research was established composed of the following members: Chairman, W. M. Corse, Monel Metal Products Corporation, Bayonne, N. J., Lt. Commander William Campbell, Columbia University, G. H. Clamer, First Vice-President and Secretary, The Ajax Metal Company, Philadelphia, H. W. Gillett, Chief Alloy Chemist, Department of the Interior, Bureau of Mines, Ithaca, Zay Jeffries, Lynite Laboratories of the Aluminum Castings Company, Cleveland, Jesse L. Jones, Westinghouse Electric Company, Paul D. Merica, The International Nickel Company, Bayonne, Paul E. McKinney, U. S. Navy Yard, John A. Mathews, President, Halcomb Steel Company, Syracuse, and lately Vice-President of the Crucible Steel Company, C. H. Mathewson, Sheffield Scientific School, Yale University, Richard Moldenke, Watchung, New Jersey, William B. Price, Waterbury, Connecticut, George C. Stone, Chief Metallurgist, The New Jersey Zinc Company, New York City, John F. Thompson, International Nickel Company, New York City, W. R. Webster, Vice-President, Bridgeport Brass Company, Bridgeport, Albert E. White, Consulting Metallurgical Engineer, Professor of Chemical Engineering, University of Michigan.

Division of Engineering.—Additional members have been appointed as follows: To represent the Institute of Mining and Metallurgical Engineers, Allen H. Rogers, New York City; to represent the American Society of Heating and Ventilating Engineers, John R. Allen, Dean of the School of Engineering and Architecture, University of Minnesota; and as a member at large representing the Association of State Highway Officials, T. R. Agg, Iowa State College.

The following committee appointments have been made: A. E. Kennelly, Harvard University, to be Chairman of the Committee on Core

Losses; T. R. Agg, Iowa State College, to be Chairman of the Committee on Economic Theory of Highway Improvement; A. N. Goldbeck, Engineer of Tests, Bureau of Public Roads, to be Chairman of the Committee on Structural Design of Roads; H. S. Mattimore, Engineer of Tests, Pennsylvania State Highway Department, to be Chairman of the Committee on Tests and Properties of Road Materials and their Combinations; J. V. Emmons, Cleveland Twist Drill Company, R. P. Devries, Lublum Steel Company, E. L. French, Crucible Steel Company of America, Henry M. Howe, Bedford Hills, New York, C. M. Johnson, Crucible Steel Company of America, Frank Lounsberry, Atlas Crucible Steel Company, A. F. MacFarland, Vanadium Alloys Company, J. A. Mathews, Crucible Steel Company of America, and Howard Scott, Bureau of Standards, to be members of the Committee on High Speed Steels.

Division of Biology and Agriculture.—A Committee on Eugenics to make arrangements for the International Eugenics Congress in New York in 1921 has been appointed, composed of the following members: L. F. Barker, Johns Hopkins Hospital, representing Medicine, E. A. Hooton, Peabody Museum, Cambridge, representing Anthropology, R. M. Yerkes, National Research Council, representing Psychology, Stewart Paton, Princeton University, representing Psychiatry, Raymond Pearl, School of Hygiene and Public Health, Johns Hopkins University, representing Eugenics and Biometry, and Alexander G. Bell, Washington.

Committees on Coöperation have been appointed by the following Societies to coöperate with the Division of Biology and Agriculture and to represent the Societies in the Division in the same manner as the present Forestry Committee: American Society of Naturalists, American Society of Agronomy, American Society of Zoölogists, and Botanical Society of America.

Division of Geology and Geography.—Additions to the Committee on Sedimentation have been made as follows: G. D. Louderback, University of California, H. E. Merwin, Geophysical Laboratory, W. H. Twenhofel, University of Wisconsin, representing the Association of State Geologists, and J. A. Udden, Director, Bureau of Economic Geology and Technology, University of Texas.

Division of Physical Sciences.—Dr. Augustus Trowbridge, Professor of Physics, Princeton University, has been made Vice-Chairman of the Division of Physical Sciences.

The following have been appointed as members of the American Geophysical Union: D. L. Hazard, U. S. Coast and Geodetic Survey, J. A. Fleming, Department of Research in Terrestrial Magnetism, S. J. Mauchly, Department of Research in Terrestrial Magnetism, T. A. Jaggar, Hawaiian Volcano Observatory, U. S. Weather Bureau, F. A. Perret, Posillipo, Naples, W. M. Davis, Sturgis-Hooper Professor of Geology, emeritus,

Harvard University, and George P. Paine, Blue Hill Observatory, Harvard University.

Division of Anthropology and Psychology.—The resignation of Dr. Franz Boas from membership in the Division has been accepted.

The following committees have been appointed:

Advisory Committee on Problems of Military Psychology: Chairman, Col. W. D. Scott, Northwestern University and the Scott Company, Philadelphia, Major G. F. Arps, Ohio State University, Major C. S. Yoakum, Carnegie Institute of Technology.

Committee on Anthropological and Psychological Study of Peoples of the United States: Chairman, *ex-officio*, Dr. W. V. Bingham, Professor of Psychology, and Director of the Division of Applied Psychology, Carnegie Institute of Technology, and Chairman of the Division of Anthropology and Psychology, National Research Council, Dr. E. L. Thorndike, Professor of Educational Psychology, Columbia University, and Dr. Clark Wissler, Curator, Department of Anthropology, American Museum of Natural History.

Committee to Formulate a Program of Research on Accumulated Army Data: Chairman, Dr. R. M. Yerkes, Chairman, Research Information Service, National Research Council, Dr. W. V. Bingham, Professor of Psychology, and Director of the Division of Applied Psychology, Carnegie Institute of Technology, and Dr. Clark Wissler, Curator, Department of Anthropology, American Museum of Natural History.

Committee on Specific Projects Outside the United States: Chairman, Dr. J. Walter Fewkes, Bureau of American Ethnology, Smithsonian Institution, Dr. William C. Farabee, Curator, University Museum, University of Pennsylvania, and Dr. R. B. Dixon, Professor of Anthropology, Harvard University.

Committee on Personnel in Research and Industry: Chairman, Beardsley Ruml, The Scott Company, Philadelphia, and Dr. James R. Angell, Dean of the Faculties of Arts, Literature and Science, and Head of the Department of Psychology, University of Chicago, and Chairman, National Research Council.

REPORT OF THE MEETINGS OF THE INTERNATIONAL RESEARCH COUNCIL AND OF THE AFFILIATED UNIONS HELD AT BRUSSELS, JULY 18–28, 1919

Presented to the Division of Foreign Relations, National Research Council

By W. W. CAMPBELL, *Chairman*, American Delegation to the International Research Council

In response to the call of the Executive Committee of the International Research Council, the third conference of representatives of the sciences in the allied and associated nations was held in Brussels on July 18 to July 28, inclusive. The purpose of the conference was to complete the organization of the International Research Council, and to form international unions, affiliated with the Council, representing the individual sciences. Duly authorized delegates from the following countries took part in the conference: Belgium, Canada, France, Great Britain and Ireland, Italy, Japan, New Zealand, Poland, Roumania, Serbia, and the United States of America. The total number of delegates from the eleven countries was slightly over two hundred. The American delegation, formed under the auspices of the National Research Council, numbered twenty-one, and was as follows:

International Research Council

W. W. CAMPBELL, *Chairman*, Chairman of American Delegation to International Astronomical Union.

C. E. MENDENHALL, Scientific Attaché at London, also representing Physics.

H. S. WASHINGTON, Scientific Attaché at Rome, also representing Geology.

MAJOR WM. BOWIE, Chairman of American Delegation to International Geophysical Union.

E. W. WASHBURN, Chairman of American Delegation to International Chemical Union.

W. S. THAYER, Representing Medicine.

JOHN C. PENNY, Representing Patents.

MAJOR DOUGLAS W. JOHNSON, Representing Geography.

Astronomical Union

W. W. CAMPBELL, *Chairman*
W. S. ADAMS
BENJAMIN BOSS
MAJOR PHILIP FOX
S. A. MITCHELL
FRANK SCHLESINGER
C. E. ST. JOHN
F. H. SEARES
JOEL STEBBINS

Geophysical Union

MAJOR WM. BOWIE, *Chairman*
L. A. BAUER
G. W. LITTLEHALES
ADMIRAL EDWARD SIMPSON

Chemical Union

E. W. WASHBURN, *Chairman*
F. G. COTTRELL
CHAS. L. PARSONS
H. S. WASHINGTON

Medicine

W. S. THAYER, *Chairman*

Geology

H. S. WASHINGTON, *Chairman*

Geography

MAJOR DOUGLAS W. JOHNSON, *Chairman*

Physics

C. E. MENDENHALL, *Chairman*

Patents

JOHN C. PENNY, *Chairman*

All sessions of the Council and of the affiliated unions and all of the related committee meetings were held in the splendid Palais des Académies, under the patronage of the Royal Belgian Academy of Sciences. The Palais, spacious in plan and sumptuous in appointments, occupies an isolated and commanding position *vis à vis* the Palais-Royal and the great Parc of the city.

Illness prevented the attendance of Professor Picard, Chairman of the Executive Committee of the Council. This position was filled for the occasion by Professor Lacroix, who presided admirably over the sessions of the Council and of the Executive Committee. Professor Hale, member of the Committee, was unable to attend the Conference, and Mr. Campbell served in his stead. Professor Schuster, Secretary of the Royal Society and General Secretary of the Executive Committee, served ably as the secretary of the Council. Other members of the Executive Committee were M. Lecointe, Director of the Royal Observatory of Belgium, and Professor Volterra, of the University of Rome.

The opening session of the Conference was honored by the presence of His Majesty, Albert, King of the Belgians, who occupied the royal box in the great assembly room. He had previously received the members of the Executive Committee in an adjoining room. An address of congratulation to the King was read by M. Hermignie, Belgian Minister of Sciences and Arts, and a cordial welcome to the delegates was pronounced by the Vice-President of the Class of Sciences of the Royal Belgian Academy. Upon the motion of the President of the Belgian Academy, the Executive Committee, as described above, served as the administrative Bureau of the Research Council throughout the Conference.

The several plenary sessions of the Research Council were devoted largely to the formulation of a Constitution which should define the purposes and govern the proceedings of the Council. The Executive Committee in special session at Paris in May had drawn up a proposed constitution, and this, in printed form, served as the basis for our deliberations. The modifications were accomplished by general consent, or by oral ballot in which each country had one vote, and final adoptions rested also upon this form of ballot.

The Conference was able to complete the structure of the Constitution, to the satisfaction of all the delegations, except in one particular: that

relating to the number of members composing the Executive Committee of the Council, and certain possible conditions governing their choice. For example, many delegates were apparently in favor of the proposal that each affiliated union should have one representative on the Executive Committee of the Council, either *ex-officio* or by the choice of the union. There was not time to consider the probable effects of the various proposals, and the Committee was fixed temporarily at five members, with the understanding that this provision should be subject to discussion and modification at the next general meeting of the Council, in 1922.

The Constitution as adopted was in French text, and it was formally agreed that interpretations of its provisions should be based upon that text. The Constitution follows herewith, the text being that issued from the office of the General Secretary of the I. R. C.

INTERNATIONAL RESEARCH COUNCIL

Statutes of Convention

I—Objects of International Council and Conditions of Admission

1. The purpose of the International Research Council is:

(*a*) To coördinate international efforts in the different branches of science and its applications.

(*b*) To initiate the formation of international Associations or Unions deemed to be useful to the progress of science in accordance with Article I of the resolutions adopted at the Conference of London, October, 1918.[1]

(*c*) To direct international scientific activity in subjects which do not fall within the purview of any existing international associations.

(*d*) To enter through the proper channels into relation with the Governments of the countries adhering to the International Research Council in order to promote investigations falling within the competence of the Council.

II—Domicile

2. The legal domicile of the International Research Council shall be at Brussels where the General Assemblies will be held and the archives kept.

Donations and legacies will be received and administered according to Belgian law.

III—Conditions of Admission

3. The countries in the following list may participate in the foundation of the International Research Council, and of any scientific Union connected with it, or join such Union at a subsequent period:

Belgium, Brazil, United States, France, the United Kingdom of Great Britain and Ireland, Australia, Canada, New Zealand, South Africa, Greece, Italy, Japan, Poland, Portugal, Roumania, Serbia.

[1] "That it is desirable that the nations at war with the Central Powers withdraw from the existing conventions relating to International Scientific Associations in accordance with the Statutes or Regulations of such Conventions, respectively, as soon as circumstances permit; and

"That new associations, deemed to be useful to the progress of science and its applications, be established without delay by the nations at war with the Central Powers with the eventual coöperation of neutral nations."

After a Union[1] has been formed, nations not included in the above list, but fulfilling the conditions of Article 1 of the resolutions of the Conference of London, may be admitted, either at their own request, or on the proposal of one of the countries already belonging to the Union.

Such requests or proposals shall be submitted to a vote of the Union concerned. A favorable majority of not less than three-quarters of the countries already forming part of the Union shall be required for admission.

4. A country may join the International Research Council, or any Union connected with it, either through its principal Academy, its National Research Council, some other national institution or association of institutions, or through its Government.

5. The Statutes of the Unions formed by the International Research Council require the approval of this Council.

IV—Administration of the Council

6. The work of the Council is directed by the General Assembly, consisting of delegates appointed by the adhering countries.

7. There shall be an Executive Committee, consisting of five members elected by the General Assembly, which shall direct the affairs of the Council in the interval between the meetings of the General Assembly in accordance with the resolutions adopted at the previous meeting of the Assembly.[2]

8. The Bureau of the Executive Committee shall consist of a President, two Vice-Presidents, and a General Secretary, who shall be elected by the General Assembly, and hold office until the end of the second General Assembly following that of their election, except that the President and one of the two Vice-Presidents shall retire at the end of the first General Assembly following that of their election. Retiring members are re-eligible.

9. The General Secretary shall be responsible for the conduct of the correspondence, the administration of funds, and the preparation and distribution of the publications authorized by the General Assembly.

10. The Executive Committee may fill up by co-optation any vacancy occurring in its body in the interval between two General Assemblies. Any person so appointed shall hold office until the next General Assembly, which shall then proceed to an election. The member thus elected shall complete the term of office of the person whose place he takes.

V—Powers and Functions of the Executive Committee

11. The Executive Committee may in the interval between two meetings of the General Assembly submit proposals to the organizations adhering to the Council for their approval. It shall do so when requested by one-third of the bodies constituting the Council.

12. The Executive Committee may nominate special Committees for the discussion of any question falling within the purview of the International Research Council. The members of such Committees need not necessarily have been delegates at one of the General Assemblies. These special Committees may themselves co-opt other members by a majority of two-thirds.

13. The Executive Committee shall present an Annual Report to the organization in each country which has joined the Council. Such reports shall also be sent to each delegate who was present at the previous meeting of the General Assembly.

[1] The term "Union" in this and subsequent paragraphs is intended to include "Association."

[2] Articles 7 and 8 are subject to revision. The Executive Committee elected in 1919 will vacate office at the next General Assembly.

VI—MEETINGS OF THE GENERAL ASSEMBLY

14. An ordinary meeting of the General Assembly shall, as a rule, take place once every three years. The date of the meeting, unless determined by the General Assembly at its previous meeting, shall be fixed by the Executive Committee, and communicated to the organizations adhering to the International Research Council at least four months beforehand, together with the agenda of business to be transacted.

15. For special reasons the President may, with the consent of the Executive Committee, summon an extraordinary meeting of the General Assembly. He is obliged to do so at the request of one-third of the adhering countries.

16. The delegates of the different countries taking part in the meetings of the General Assembly shall be appointed by the organizations adhering to the International Research Council.

17. The President of the Executive Committee may invite to a meeting of the General Assembly scientific men who are not delegates, provided that they are subjects of one of the adhering countries. Such invited guests may take part in the discussions, but shall have no power of voting. Members of any of the Special Committees mentioned in Article 12 shall have the right to attend those meetings of the General Assembly which deal with the subjects referred to the Committee.

18. The agenda of business to be transacted at a meeting shall be determined by the Executive Committee, and communicated to the adhering organizations at least four months before the first day of the meeting. No question which has not been placed on the agenda shall be discussed, unless a proposal to that effect be approved by not less than one-half of the total votes of the countries represented at the General Assembly.

VII—FINANCE AND VOTING POWER

19. The Executive Committee shall prepare an estimate of the budget for each of the years intervening between two General Assemblies. A Finance Committee, nominated for the purpose by the General Assembly, shall examine this estimate, together with the accounts for the preceding years. This Committee shall submit separate reports on the two questions to the General Assembly which, having considered these reports, shall fix the unit of contribution for the succeeding period. The contributions due from countries and their corresponding voting powers are determined by the population of the countries according to the following scale:

POPULATION	NO. OF VOTES	NO. OF UNITS OF CONTRIBUTION
Less than 5 millions.....	1	1
Between 5 and 10 millions.....	2	2
Between 10 and 15 millions.....	3	3
Between 15 and 20 millions.....	4	5
Over 20 millions.....	5	8

Each state is at liberty to include in its population, according to the indication of its Government, the inhabitants of its colonies or protectorates. Self-governing dominions (South Africa, Australia, Canada, New Zealand) have separate voting power according to the above scale.

The contribution to be levied during the first period of the Convention shall not exceed 250 francs[1] per annum per unit.

In each country the organization adhering to the Council shall be responsible for the payment of the contribution.

[1] The value of the franc shall be that of the French currency.

20. The income of the Council derived from the contributions of the different countries shall be devoted to the expenses of the Bureau.

Funds derived from donations shall be used by the Council in accordance with the wishes expressed by the donors.

If any country withdraws from the Council, it resigns at the same time its rights to a share in the assets of the Council.

21. At the General Assembly votes upon scientific questions shall be decided by the majority of votes cast by the delegates present. In questions of an administrative character, and all questions not purely scientific, the vote shall be taken by countries, each country having the number of votes indicated in Article 19. In case of doubt as to the category to which a question belongs, the Chairman of the meeting shall decide. Votes taken in Committees shall be counted individually and not by countries. When there is an equal division of votes, the Chairman shall in all cases have a second or casting vote.

22. In questions of an administrative character, any country which is not represented by a delegate may forward its vote to the President by post, and such votes shall be counted if received before the vote is taken.

VIII—DURATION OF THE CONVENTION AND MODIFICATIONS

23. The present Convention shall come into force on the 1st of January, 1920, provided that at least three of the countries mentioned in Article 3 have signified their adhesion. It shall remain in force until the 31st of December, 1931, and shall then, with the assent of the adhering countries, be continued for a further period of 12 years.

24. No change shall be made in the terms of the Convention except with the approval of two-thirds of the votes of the adhering countries.

25. The French text of the Convention (of which the above is a translation) shall be considered to be the authoritative text.

At the final session of the Council the original Executive Committee was unanimously re-elected, to serve during the periods specified in the Constitution. The Committee consists of:

Professor Emile Picard, *President*
Professor Arthur Schuster, *General Secretary*
Professor George E. Hale, *Member*
Director Georges Lecointe, *Member*
Professor Vito Volterra, *Member*

To secure overlapping in the terms of service, and, therefore, continuity of administration, the first president and one member of the first Executive Committee are to hold office for three years only, but the General Secretary and the other two members are to serve during six years. Officers and members elected at the next triennial meeting, or at later meetings, are to serve during six years. All officers and members are re-eligible.

The duration of the present convention, as defined by the Constitution, is twelve years, to December 31, 1931. It may then be renewed for another period of twelve years, with the assent of the adhering countries.

Sixteen allied and associated countries are named in the Constitution as eligible to participate in the foundation of the Research Council. The five of these countries not represented by delegates may become members of the Council and of the affiliated unions by expressing their adhesion

to the terms of the Constitutions and by pledging their corresponding shares of financial support.

At the concluding session of the Conference, the Executive Committee was authorized to invite all leading neutral countries to enter into full membership relations with the Research Council and the affiliated unions.

In accordance with the terms of the Constitution, each adhering country is under obligations to contribute to the financial support of the Council. The relative amount of its contribution and the number of votes to which it is entitled are as defined by the following scale:

POPULATION OF COUNTRY	NUMBER OF VOTES	NO. OF FINANCIAL UNITS
Less than 5 million inhabitants.....	1	1
Between 5 and 10 million inhabitants...	2	2
Between 10 and 15 million inhabitants...	3	3
Between 15 and 20 million inhabitants...	4	5
More than 20 million inhabitants.....	5	8

The value of the financial unit for the Research Council itself during the first period of the convention (three years) may not exceed 250 francs per annum. That is, the annual obligation of the United States, in the first period, may not exceed 2000 francs annually. Funds received in this manner from the adhering countries shall be devoted to meeting the expenses of the administrative Bureau of the Council.

In advance of each triennial meeting the Executive Committee shall prepare a proposed financial budget for each of the three years following the meeting. A Finance Committee, nominated by the General Assembly of the Council, shall study this budget, and likewise audit the financial accounts for the preceding period. This Committee shall submit to the General Assembly separate and distinctive reports upon these two subjects. The Council in General Assembly shall determine the value of the contributive financial unit.

It was decided that the city of Brussels shall be the legal seat of the International Council of Research, and that the proceedings shall be governed by the laws of Belgium. It was likewise determined that the administrative seat of the Council—the Secretariat—shall be in the city where resides the General Secretary, in this instance in London.

The Executive Committee of the Council will submit the Constitution to the national academies, the national research councils, or the governments whose representatives took part in the Brussels Conference, for their approval. The International Research Council shall be considered as definitively constituted when three of the greater allied countries shall have expressed their approval. It is hoped that these approvals may be obtained not later than January 1, 1920.

One of the fundamental objects of the International Research Council is to encourage the creation of international unions representing, respectively, the chief divisions of science, in so far as such unions may be considered desirable and necessary. Simultaneously with the completion of the organization of the Research Council in Brussels three affiliated unions were definitively established, as follows:

The International Astronomical Union.

The International Geodetic and Geophysical Union.

The International Union of Pure and Applied Chemistry.

A Union of Biological Sciences and a Union of Scientific Radio-Telegraphy were organized, though somewhat less definitively; constitutions were formulated at Brussels, but it was decided that the plans proposed should be submitted to the different academies for their consideration.

Proposals were made and carefully considered for the establishing of unions of the mathematical sciences, of the physical sciences, of the geological sciences, of the geographical sciences, of bibliography, etc., but organic decisions were not reached. The expectation is that these unions will take definite form at or before the time of the next triennial meeting, in 1922. The mathematicians have decided, for example, to meet in Strasbourg in the year 1920, and the subject of an international union will at that time receive their consideration.

Engineering science and patent laws, both from the international point of view, were considered in Brussels by delegates representing these subjects.

A detailed report on the organization of the International Astronomical Union will have been presented to the National Research Council by the Secretary of the American Section.

The Secretary of the American Section of the Geodetic and Geophysical Union will have presented to the National Research Council a detailed report on the organization of the Union.

The Secretary of the American Section of the International Union of Pure and Applied Chemistry will have presented to the National Research Council a detailed report concerning the Union.

The International Union of Biological Sciences made progress in the direction of definitive organization. It is proposed that this Union shall be composed of six sections, as follows: General Biology, Physiology, Zoölogy, Botany, The Medical Sciences, Applied Biology.

The International Union of Radio-Telegraphy, which made progress toward definitive organization under the able leadership of General Ferrié, Inspector General of the Service of Military Telegraphy, Paris, proposes to concern itself chiefly with the study of radiometric phenomena as a pure science.

The activities of each union require that financial resources be available, as in the case of the International Research Council. It is proposed

that all the contributions be united for any one country in one appropriation, payable to the International Research Council, which shall apportion the funds in accordance with the relative needs. The wisdom of this policy can scarcely be questioned, in view of the intimate relationship of governments to the subject of appropriations for these purposes. A definite statement as to the total financial obligations of the United States cannot now be made. This will depend in part upon the list of countries which give their adherence to the proposals. The total number of contributory financial units remains unknown for the present. It is hoped that the number depending upon the allied and associated countries will be augmented by the adherence of many neutral countries. It should also be stated that the effective population of certain member countries, and, therefore, the amounts of their financial contributions, will depend upon whether they decide to include the inhabitants of their colonies and protectorates.

A resolution expressing the profound sympathy of the delegates for their colleagues in Russia was adopted unanimously at the closing session of the Conference.

Formal receptions were tendered to the delegates: by the city of Brussels, represented by the celebrated Burgomaster Adolf Max and the members of his Council, at the Hotel de Ville; by M. Hermignie, Minister of Sciences and Arts, and Madame Hermignie; and by M. Hymans, Minister of Foreign Affairs, and Madame Hymans. M. Lecointe, Director of the Belgian Royal Observatory at Uccle (a suburb of Brussels) and Madame Lecointe, received the astronomical, geodetic, and geophysical delegates in the director's residence. The members of the Observatory staff kindly provided opportunities for the delegates to see the more interesting and important features of the Observatory and its work. Much of the success of the Brussels Conference was due to the thoughtful and able services of Director Lecointe, Belgian member of the Executive Committee of the Research Council.

The Conference was memorable for the splendid spirit of coöperation and personal friendliness which prevailed universally. The discussions were conducted with sincere efforts to reach wise conclusions. The decisions were in general unanimous. These facts are happy indications that the evolution of the new organizations will follow along successful lines.

The chairman desires to express his appreciation of the uniformly helpful spirit in which every member of the American delegation joined in the solution of the many problems before the Conference.

Respectfully submitted,

W. W. CAMPBELL,

Chairman of the American Delegation to the International Research Council.

REPORT ON THE ORGANIZATION OF THE INTERNATIONAL ASTRONOMICAL UNION[1]

Presented for the American Section, International Astronomical Union

BY W. W. CAMPBELL, *Chairman*, AND JOEL STEBBINS, *Secretary*

The American delegation to the conference of interallied academies held in London in October, 1918, had proposed that the existing organizations relating internationally to one and the same subject, to Astronomy for example, should be succeeded by a single society so formulated as to serve the purposes of all the associations from which they should withdraw. This policy was approved by the conference. At meetings held later in Paris it was decided that representatives of Astronomy in the allied and associated nations should meet in Brussels in July, 1919, to create the International Astronomical Union.

The American delegation of astronomers was organized under the auspices of the National Research Council. The Council proposed that the delegates be chosen by a subsidiary organization known as the American Section of the (proposed) Astronomical Union; this Section for the present occasion to be constituted as follows:

National Academy of Sciences—5 members: W. W. CAMPBELL, GEORGE E. HALE, A. A. MICHELSON, F. R. MOULTON, FRANK SCHLESINGER.

American Astronomical Society—20 members: C. G. ABBOT, W. S. ADAMS, R. G. AITKEN, S. I. BAILEY, E. E. BARNARD, L. A. BAUER, BENJAMIN BOSS, E. W. BROWN, W. S. EICHELBERGER, E. B. FROST, J. F. HAYFORD, W. J. HUMPHREYS, W. J. HUSSEY, A. O. LEUSCHNER, S. A. MITCHELL, H. N. RUSSELL, C. E. ST. JOHN, F. H. SEARES, V. M. SLIPHER, JOEL STEBBINS.

American Mathematical Society—3 members: G. D. BIRKHOFF, W. D. MACMILLAN, R. S. WOODWARD.

American Physical Society—3 members: J. S. AMES, HENRY CREW, THEODORE LYMAN.

U. S. Coast and Geodetic Survey—1 member: WILLIAM BOWIE.

U. S. Naval Observatory—1 member: J. A. HOOGEWERFF.

The members selected later by these organizations are as listed above.

The American Section thus constituted held its first meeting in the office of the National Research Council, Washington, D. C., on March 8, 1919. Twenty of the thirty-three members were present. Mr. W. W. Campbell was appointed permanent Chairman of the Section, and Mr. Joel Stebbins, Secretary. An Executive Committee of the Section was appointed, consisting of the Chairman, the Secretary, and Messrs. C. G. Abbot, E. W. Brown, and Frank Schlesinger. The Section gave assent to the acts of the London Conference, and formally approved the proposal that those nations which had remained neutral throughout the war should be admitted into the International Astronomical Union promptly upon

[1] Reprinted in Reprint and Circular Series of the National Research Council, Number 10.

the conclusion of peace. The Section did not attempt to outline the form of an international organization which would promise the greatest usefulness, but concerned itself largely with the technical work of the proposed Union in which the American astronomers would be chiefly interested. The Executive Committee was instructed to appoint subcommittees of the American Section which should prepare reports upon nineteen prominent divisions of astronomical research, for presentation at a later meeting of the Section. The Brussels delegation was selected in part, and the Executive Committee was authorized to complete it.

The delegation was eventually composed as follows:

W. W. CAMPBELL, Lick Observatory, *Chairman;* W. S. ADAMS, Mount Wilson Observatory; BENJAMIN BOSS, Dudley Observatory, Albany; MAJOR PHILIP FOX, Dearborn Observatory, Evanston, at that time with the American Expeditionary Forces in France; S. A. MITCHELL, McCormick Observatory, Virginia; C. E. ST. JOHN, Mount Wilson Observatory; F. H. SEARES, Mount Wilson Observatory; FRANK SCHLESINGER, Allegheny Observatory, Pittsburgh; JOEL STEBBINS, University of Illinois, *Secretary.*

The special subcommittees were in due time constituted, as follows:

Committee on the Variation of Latitude.—F. B. LITTELL, *Chairman,* A. O. LEUSCHNER, FRANK SCHLESINGER.

It was voted to ask the American Section of the International Geophysical Union to appoint a similar committee to confer and make a joint recommendation on the most promising method of continuing the variation of latitude work.

Committee on Standards of Wave-Length.—HENRY CREW, *Chairman,* H. D. BABCOCK, KEIVIN BURNS, W. W. CAMPBELL, C. E. ST. JOHN.

Committee on Solar Rotation.—C. E. ST. JOHN, *Chairman,* W. S. ADAMS, FRANK SCHLESINGER.

Committee on Eclipses.—S. A. MITCHELL, *Chairman,* E. E. BARNARD, H. D. CURTIS.

Committee on Stellar Classification.—H. N. RUSSELL, *Chairman,* W. S. ADAMS, MISS ANNIE J. CANNON, R. H. CURTISS.

Committee on Asteroids and Comets.—A. O. LEUSCHNER, *Chairman,* E. W. BROWN, G. H. PETERS.

Committee on Almanacs.—W. S. EICHELBERGER, *Chairman,* E. W. BROWN, R. H. TUCKER.

Committee on Radial Velocities.—W. W. CAMPBELL, *Chairman,* W. S. ADAMS, J. S. PLASKETT.

Committee on Double Stars.—R. G. AITKEN, *Chairman,* ERIC DOOLITTLE, W. J. HUSSEY.

Committee on Notation, Units, and Economy of Publication.—W. J. HUMPHREYS, *Chairman,* E. B. FROST, A. O. LEUSCHNER.

Committee on Meridian Astronomy.—BENJAMIN BOSS, *Chairman,* F. B. LITTELL, FRANK SCHLESINGER.

Committee on Abstracts and Bibliographies.—F. E. FOWLE, *Chairman,* H. D. CURTIS, G. S. FULCHER.

Committee on Research Surveys.—G. E. HALE, *Chairman,* F. R. MOULTON, HARLOW SHAPLEY.

Committee on Stellar Photometry.—F. H. SEARES, *Chairman,* S. I. BAILEY, F. C. JORDAN, J. A. PARKHURST, JOEL STEBBINS.

Committee on Wireless Determination of Longitude.—J. A. HOOGEWERFF, *Chairman,* W. W. CAMPBELL, J. J. CARTY. This committee was requested to study the feasibility

of determinations of longitude by wireless at widely distributed stations, and report on what seems to be the proper time and method for such undertakings.

Committee on Solar Radiation.—MR. C. G. ABBOT was asked to prepare a report on solar radiation.

Committee on the Spectroheliograph.—The Mount Wilson Observatory was asked to prepare a report on work with the spectroheliograph.

Committee on Reform of the Calendar.—R. T. CRAWFORD, *Chairman*, W. W. CAMPBELL, HAROLD JACOBY.

Another subcommittee was appointed at a later date, as follows:

Committee on Stellar Parallaxes.—FRANK SCHLESINGER, *Chairman*, W. S. ADAMS, S. A. MITCHELL.

The American Section met again in the office of the National Research Council on June 23 and 24, to receive and act upon the reports of the technical subcommittees, to consider questions of policy, and to instruct the delegation which would represent the Section at Brussels. Twenty members of the Section were present. The reports of the subcommittees, presenting the aspects of their subjects which were then prominent in the minds of astronomers, and suggesting promising procedure for the immediate future, were of a high order of excellence. They are appended to this report in the form in which they were adopted by the Section. The discussions, conducted with enthusiasm and frankness, were extremely valuable.

A joint meeting of the American Section with the proposed International Geodetic and Geophysical Union (Major William Bowie, Chairman) was held on June 24.

The delegation of astronomers sailed from New York on June 30 and reached London on July 7. The next ten days were devoted profitably and with unusual pleasure to renewing old scientific acquaintances and making many new ones, to learning of progress in astronomy and the related sciences made in Great Britain during the war period, to discussing with our British colleagues the many problems coming up for consideration at the Brussels Conference, etc.

The President and Council of the Royal Astronomical Society had most kindly arranged by cable that the members of the American delegation should attend and address a special meeting of the Society on the afternoon of July 11. At this meeting, under the presidency of Professor Fowler, each member of our astronomical delegation, and likewise Dr. L. A. Bauer, Secretary of the Geophysical Section, addressed the Society informally on the scientific subject which at that time especially interested him. This meeting had been preceded by a reception in the rooms of the Society, which gave opportunity to meet many British men and women whose names had long been familiar.

Opportunities to observe the equipment and work of the Royal Observatory at Greenwich, of the University observatories at Cambridge and Oxford, and of the laboratory of Professor Fowler in South Kensington were provided by members of their staffs.

In common with many British colleagues, the American delegation journeyed from London to Brussels on July 17. The Conference lasted from July 18 to July 28, inclusive. Thirty-three astronomical delegates were present: eight from Belgium, eight from Great Britain, seven from France, one from Italy, and nine from the United States.

The most difficult question confronting the International Astronomical Union related to its composition. Should it be organized as a great astronomical society, comprehending all divisions of astronomical science in a somewhat general or indefinite manner, or should it consist of working committees, each committee concerned with some definite line of astronomical research? Should there, for example, be a great division of the Union relating to the solar system, or, on the contrary, several comparatively small committees occupying themselves responsibly with such definite subjects as solar rotation, eclipses, solar radiation, etc.? Should there be another great division of the Union relating to the sidereal system, or, on the contrary, several comparatively small committees occupying themselves responsibly with such definite subjects as stellar parallaxes, radial velocities of the stars, double stars, variable stars, etc.? The latter plan was adopted. To begin with, thirty-two such special committees were established. These vary in size in accordance with the requirements and present states of the problems concerned. The Committee on the Carte du Ciel and the Committee on Meridian Astronomy are relatively large, with more than a dozen members each, whereas other committees, such as those on Lunar Nomenclature, on Solar Radiation, on Astronomical Ephemerides, etc., are relatively small, with not more than half a dozen members each. The separate committees are working bodies. They are intended to be essentially complete in themselves. The Union as a whole will meet in general every three years in different educational centers of the world, but it is expected that the members of a given committee will find many occasions between general meetings when conferences by letter or otherwise may profitably occur. Committee reports will doubtless aim to represent the views of the committees as to the current status of their subjects, and to present practical suggestions for further undertakings. In other words, it is anticipated that each committee shall make known through its report, presented in a general session of the Union, those aspects of its subject with which the workers in that subject may most profitably acquaint and busy themselves.

Each committee has power to add to its numbers by favorable vote of two-thirds of the current membership of the committee.

It is provided in the Constitution that the committees shall automatically go out of existence at the close of each triennial meeting of the Union. This offers the opportunity for reorganization upon the basis of those who work.

The Constitution of the Union, as distributed from the office of the Gen-

eral Secretary of the International Research Council, follows this report together with the Regulations of the International Time Commission and of the International Central Bureau for Astronomical Telegrams, which were adopted by the Union.

The number of votes and the relative financial obligations of each adhering country are fixed by a scale identical with that which governs these subjects in the International Research Council. The unit of financial contribution for the United States during the first three years of the convention may not surpass 1,500 francs annually. In this connection we should note the fact that, upon the recommendation of the American delegates representing Astronomy and Geophysics, the administration of the latitude variation problem is to be transferred to the International Astronomical Union. The financial obligation of the United States in support of this work is one of the larger items entering into the financial budget of the Astronomical Union.

The budget of the Astronomical Union for subsequent years shall be determined in a manner precisely parallel to that adopted for the Research Council.

The following officers of the Astronomical Union were elected in general session:

President, M. BENJAMIN BAILLAUD, Director of the Paris Observatory.

General Secretary, ALFRED FOWLER, F.R.S., Professor of Astrophysics, Imperial College of Science and Technology, South Kensington, London.

Vice-Presidents, W. W. CAMPBELL, Director of the Lick Observatory; SIR F. W. DYSON, the Astronomer Royal, Greenwich, London; M. GEORGES LECOINTE, Director of the Royal Observatory of Belgium, Brussels; M. ANNIBALE RICCÒ, Director of the Observatory of Mt. Etna, Sicily.

The fifth vice-presidency was left vacant for the present, pending the adhesion of countries which had remained neutral during the war. The officers here listed compose the Executive Committee of the Union.

The technical committees are as follows:

1. *Committee on Relativity.*—EDDINGTON, *Chairman*, ARMELLINI, BRILLOUIN, H. D. CURTIS, DE DONDER, ISHIWARA, JEANS, LEVI-CIVITA, E. PICARD.

2. *Committee on the Republication of Early Astronomical Works.*—DREYER, *Chairman*, BIGOURDAN, BIRKENMAYER, CREW, KNOBEL, MASCART, TERAO.

3. *Committee on Notation, Units, and Economy of Publication.*—STROOBANT, *Chairman*, BIGOURDAN, BOSS, DESLANDRES, DYSON, FROST, TURNER.

4. *Committee on Ephemerides.*—COWELL, *Chairman*, ANDOYER, BROWN, DYSON, EICHELBERGER.

5. *Committee on Abstracts and Bibliography.*—BAILLAUD, *Chairman*, BIGOURDAN, H. D. CURTIS, FOWLER, KNOBEL, MILLOSEVICH, STROOBANT.

6. *Committee on Astronomical Telegrams.*—LECOINTE, *Chairman*, BAILEY, BAILLAUD.

7. *Committee on Dynamical Astronomy and Astronomical Tables.*—ANDOYER, *Chairman*, BROWN, COWELL, GLAISHER, HAMY, LEUSCHNER, MOULTON, SAMPSON.

8. *Committee on Meridian Astronomy (including Refraction).*—HOUGH, *Chairman*, ANTONIAZZI, DE LA BAUME PLUVINEL, BIGOURDAN, BOSS, DYSON, EDDINGTON, GONNESSIAT, HAMY, DI LEGGE, LITTELL, PHILIPPOT, PORTER, TUCKER, (Australian).

9. *Committee on Theoretical and Applied Optics and the Study of Instruments.*—Hamy, *Chairman*, Carty, Conrady, Cotton, Delvosal, Fabry, Littell, Lunn, Sampson.

10. *Committee on Solar Radiation.*—Abbot, *Chairman*, Bemporad, Callendar, Deslandres, Fowle, Gorczynski.

11. *Committee on the Spectro-Helio-Enregistreur.*—Deslandres, *Chairman.*

12. *Committee on the Solar Atmosphere.*—Hale, *Chairman*, Deslandres, Evershed, Fowler, Hirayama, Lee, Newall, Pérot, Riccò, St. John.

13. *Committee on Astronomical Expeditions, Eclipses,' etc.*—de la Baume Pluvinel, *Chairman*, Angot, Barnard, Campbell, Deslandres, Fowler, Hinks, Mitchell, Palazzo, Riccò.

14. *Committee on the Determination of Wave-Lengths and Tables of the Solar Spectrum.*—St. John, *Chairman*, Babcock, Fabry, Fowler, Hamy, Meggers, Merton, Nagaoka, Newall, Pérot, Wright.

15. *Committee on Solar Rotation.*—Newall, *Chairman*, Adams, Deslandres, Evershed, DeLury, Pérot, Sampson, St. John.

16. *Committee on Physical Observations of the Planets.*—Cerulli, *Chairman*, Barnard, Deslandres, Jarry-Desloges, Lampland, Mascart, Phillips, Stroobant, H. Thomson.

17. *Committee on Lunar Nomenclature.*—Turner, *Chairman*, Bigourdan, Miss Blagg, Pickering, Puiseux.

18. *Committee on the Determination of Terrestrial Longitude by Wireless Telegraphy.*—General Ferrié, *Chairman*, Bigourdan, Carty, Delporte, Dyson, Hamy, Hoogewerff, Hough, Nakano, Sampson.

19. *Committee on the Variation of Latitude.*—Kimura, *Chairman*, de la Baume Pluvinel, Bianchi, Bigourdan, Brillouin, Celoria, Hills, Jeffreys, Jones, Larmor, Littell, Reina, Ross, Schlesinger, Volta.

20. *Committee on Asteroids.*—Brown, *Chairman*, Ant. Abetti, Andoyer, Bigourdan, Bourget, Cowell, Crommelin, Fayet, Gonnessiat, Krassowski, Leuschner, Metcalf, Millosevich, Stroobant.

21. *Committee on Comets.*—Leuschner, *Chairman*, Andoyer, de la Baume Pluvinel, Barnard, Crommelin, Deslandres, Fayet, Fowler,

22. *Committee on Meteors.*—Denning, *Chairman*, Deslandres, Lebeuf, Mitchell, Olivier, Mrs. Wilson.

23. *Committee on the Carte du Ciel.*—Turner, *Chairman*, Baillaud, Balbi, Baldwin, Bemporad, Bhaskaran, Cook, Cosserat, Curlewis, Delvosal, Dyson, Gonnessiat, Hough, Lecointe, L. Picart, Riccò, Sampson, Schlesinger.

24. *Committee on Stellar Parallaxes.*—Schlesinger, *Chairman*, G. Abetti, Adams, Bigourdan, Cosserat, Davidson, Dyson, Fox, Miller, Mitchell, Rambaut, Van Biesbroeck.

25. *Committee on Stellar Photometry.*—Seares, *Chairman*, Baillaud, Eddington, Halm, Miss Leavitt, Melotte, Parkhurst, Sampson, Turner.

26. *Committee on Double Stars.*—Aitken, *Chairman*, G. Abetti, Bigourdan, Doolittle, Espin, Fox, Hussey, Jackson, Jonckheere, Van Biesbroeck.

27. *Committee on Variable Stars.*—Bailey, *Chairman*, Bemporad, Brook, Cosserat, Jordan, Markwick, Mascart, Pace, Phillips, Plummer, Russell, Shapley, Stebbins, Turner.

28. *Committee on the Nebulae.*—Bigourdan, *Chairman*, Deslandres, Dreyer, Fabry, Knox-Shaw, Newall, Reynolds, Slipher, Wright.

29. *Committee on the Spectral Classification of Stars.*—Adams, *Chairman*, Miss Cannon, R. H. Curtiss, Fowler, de Gramont, Hamy, Newall, Plaskett, Russell.

30. *Committee on Stellar Radial Velocities.*—Campbell, *Chairman*, Adams, Deslandres, Hamy, Lunt, Newall, Plaskett.

31. *Committee on Time.*—SAMPSON, *Chairman*, BAILLAUD, BIGOURDAN, CELORIA, DYSON, GENERAL FERRIÉ, HOOGEWERFF, HOUGH, LECOINTE, PHILIPPOT.

32. *Committee on the Reform of the Calendar.*—CARDINAL MERCIER, *Honorary Chairman*, BIGOURDAN, *Chairman*, CAMPBELL, CRAWFORD, DESLANDRES, DYSON, LECOINTE, MILLOSEVICH, RICCÒ, SAMPSON.

Certain of the committees, such as those on Eclipses, on Time, etc., held meetings in Brussels promptly following their formation. It was deeply regretted that the work of organization was so time-consuming as to prevent nearly all of the committees from holding first meetings.

The invitation of the Italian delegates that the next meeting of the International Astronomical Union should be held in Rome in the year 1922 was accepted.

The earnestness and enthusiasm with which the work of organization was conducted, and the unanimity with which decisions were finally made, are the satisfactory indications that the Union will meet efficiently its purpose of promoting astronomical research through the coördinated efforts of the astronomers in many countries.

Respectfully submitted,

W. W. CAMPBELL, *Chairman*
JOEL STEBBINS, *Secretary*
American Section, International Astronomical Union.

INTERNATIONAL RESEARCH COUNCIL

INTERNATIONAL ASTRONOMICAL UNION

Statutes of Convention

I—OBJECTS OF THE UNION AND CONDITIONS OF ADMISSION

1. The purpose of the Union is:

(*a*) To facilitate the relations between astronomers of different countries where international coöperation is necessary or useful.

(*b*) To promote the study of astronomy in all its departments.

2. The admission of countries to the Union shall be subject to the Regulations of the International Research Council.

II—NATIONAL COMMITTEES

3. A National Committee shall be formed in each of the countries belonging to the Union. It shall be formed under the responsibility of the principal Academy of the country concerned, or of its National Research Council, or of some other national institution or association of institutions, or of its Government.

4. The functions of the National Committees shall be to promote and coördinate in their respective countries the study of the various branches of Astronomy, more especially in relation to their international requirements.

They shall be empowered to propose singly or jointly with other National Committees questions falling within the purview of the Union for discussion by the Union.

The National Committees shall nominate delegates to represent them at the meetings of the Union.

III—ADMINISTRATION OF THE UNION

5. The work of the Union shall be directed by the General Assembly of the delegates.

6. The Bureau of the Union shall consist of a President, not more than five Vice-Presidents, and a General Secretary, who shall be elected by the General Assembly,

and hold office until the end of the second General Assembly following that of their election; provided that the first President and three of the first Vice-Presidents (as determined by the drawing of lots) shall retire at the end of the first General Assembly following that of their election. The retiring members are re-eligible.

This Bureau shall be the Executive Committee of the Union.

The five Vice-Presidents shall be chosen so as to represent the different branches of Astronomy.

In the case of any vacancy occurring among its members, the Executive Committee shall have power to fill up such vacancy. Any person so appointed shall hold office until the next General Assembly, which shall then proceed to an election. The member thus elected shall complete the term of office of the person whose place he takes.

There shall be an Administrative Office, which, under the direction of the General Secretary, shall conduct the correspondence, preserve the archives, prepare and issue the publications sanctioned by the Union, and administer its funds.

IV—Standing Committees

7. The General Assembly shall appoint Standing Committees, for the study of special branches of Astronomy, the encouragement of collective investigations, and the discussion of questions relating to international agreements or to standardization.

The Standing Committees shall present reports of their work to the Union.

8. The President and the members of each Committee are elected by the General Assembly on the nomination of the Executive Committee of the Union. They hold office until the end of the next ordinary General Assembly, and are re-eligible.

When a Standing Committee is composed of members appointed partly by the Astronomical Union and partly by some other Union connected with the International Research Council, it shall have the power to elect its own President.

Standing Committees shall draw up their own by-laws. They may co-opt other members by a majority of two-thirds. Such co-opted members need not necessarily be delegates, but shall be subjects of one of the countries adhering to the Union.

9. With the sanction of the Executive Committee, a Standing Committee may issue its publications independently. A Standing Committee may entrust any part of its work to any national institution or to individuals.

V—The General Assembly

10. The General Assembly shall, as a rule, hold an ordinary meeting once every three years. The date and place of the meeting, unless determined by the General Assembly at its previous meeting, shall be fixed by the Executive Committee, and communicated at least four months beforehand to the adhering organizations.

11. For special reasons, the President of the Union, with the consent of the Executive Committee, may summon an extraordinary meeting of the General Assembly. He must do so at the request of one-third of the votes of the countries adhering to the Union.

12. All members of the National Committees may attend the meetings and take part in the discussions of the General Assembly without power of voting.

The President of the Union may invite scientific men, who are not delegates, to attend a meeting of the General Assembly, provided they are subjects of one of the adhering countries. Such invited guests may take part in the discussions, but shall have no power of voting.

Members of any committees who are not delegates (Article 8) shall have the right to attend those meetings of the General Assembly which deal with the subjects referred to the Committee.

13. The agenda of business to be transacted at a meeting shall be determined by the Executive Committee and communicated to the adhering organizations at least four

months before the first day of the meeting. No question which has not been placed on the agenda shall be discussed, unless a proposal to that effect be approved by one-half of the countries represented at the General Assembly.

VI—FINANCE AND VOTING POWER

14. The Executive Committee shall prepare an estimate of the budget for each of the years intervening between two General Assemblies. A Finance Committee, nominated for the purpose by the General Assembly, shall examine this estimate together with the accounts for the preceding years. This Committee shall submit separate reports on the two questions to the General Assembly, which, having considered these reports, shall fix the unit of contribution for the succeeding period. The contributions due from the adhering countries, and their corresponding voting powers, are determined by the population of the country according to the following scale:

POPULATION	NO. OF VOTES	NO. OF UNITS OF CONTRIBUTION
Less than 5 millions.....	1	1
Between 5 and 10 millions.....	2	2
Between 10 and 15 millions.....	3	3
Between 15 and 20 millions.....	4	5
Over 20 millions.....	5	8

Each country is at liberty to include in its population the inhabitants of its colonies or protectorates, their number being that indicated by its Government. Self-governing Dominions (South Africa, Australia, Canada, New Zealand) have separate voting power according to the above scale.

The contribution to be levied during the first period of the convention shall not exceed 1,500 francs per annum per unit.[1]

In each country the organization adhering to the Council shall be responsible for the payment of the contribution.

15. The income of the Union is to be dovoted to:

(*a*) Cost of publication and expenses of administration.

(*b*) Cost of reduction and discussion of observations, including the payment of assistants.

Funds derived from donations shall be used by the Union, in accordance with the wishes expressed by the donors.

If any country withdraws from the Union it resigns at the same time its rights to a share in the assets of the Union.

16. At the General Assemblies votes upon scientific questions shall be decided by a majority of votes cast by the delegates present and voting. In questions of administrative character, and all questions not purely scientific, the vote shall be taken by countries, each country having the number of votes designated in Article 14. In case of doubt as to the category to which a question belongs, the Chairman of the meeting shall decide. Votes taken in Committee shall be counted individually and not by countries. When there is an equal division of votes, the Chairman shall in all cases have a second or casting vote.

17. In questions of an administrative character which appear on the agenda a country not represented at a meeting may forward its vote to the President by post, and such vote shall be counted if received before the vote is taken.

[1] The value of the franc shall be that of the French currency.

VII—REGULATIONS

18. The General Assembly may draw up regulations for the conduct of its business, the general duties to be assigned to the officers of the Union, and all matters not provided for in the Convention.

Each standing Committee may draw up its own regulations, which shall be approved by the General Assembly before coming into force.

No regulation shall be made, either by the General Assembly or by the Standing Committees, which violates the term of the present Convention.

VIII—DURATION OF CONVENTION AND MODIFICATIONS

19. The present Convention shall remain in force until December 31, 1931. It may then, with the assent of adhering countries, be continued for a further period of twelve years.

20. No change shall be made in the terms of the Convention except with the approval of two-thirds of the votes of the adhering countries.

21. The French text (of which the above is a translation) shall be considered to be the authoritative text.

INTERNATIONAL RESEARCH COUNCIL

INTERNATIONAL ASTRONOMICAL UNION

International Time Commission

REGULATIONS

1. An International Time Commission shall be formed for the purpose of unifying time by means of radio-telegraphic or other signals, whether these are scientific signals of great precision, or ordinary signals adapted to the needs of navigation, meteorology, seismology, railways, postal and telegraphic services, public administration, clock-makers, individuals, etc.

2. The domicile of the Commission shall be located at Paris.

3. The Bureau shall consist of a President, a Vice-President, and a General Secretary; the latter two shall be elected by the Commission itself. The General Secretary, whose tenure of office shall terminate on December 31, 1931, shall fulfil the functions of Director of the International Time Bureau.

4. The purpose of the International Time Bureau, located at the National Observatory of Paris, is:

(*a*) As regards ordinary signals, to harmonize the determinations of universal time, expressed in Greenwich Time, which shall be transmitted to them by the National Time centers. These shall be responsible for calculating, with the highest obtainable precision, the mean time, as deduced from determinations made by the Observatories of their respective countries. These results shall be sent as quickly as possible to the emitting stations and to the National Centers.

(*b*) As regards scientific signals of precision, to harmonize the time determinations made by the associated observatories, and deduce from them the most exact time.

The International Time Bureau shall publish the results of its comparisons. Any result whose publication is delayed shall be communicated in detail to all scientific associations and official institutions which may send in a request to that effect.

5. In addition to the Director, the International Time Bureau shall consist of:

(*a*) Scientific collaborators, nominated and dismissed by the Commission, on the proposal of the Director of the International Bureau. They shall be charged, with or without indemnification, with the conduct of special studies. Their term of office shall not exceed two years, but they shall be re-eligible.

(*b*) Scientific assistants, nominated and dismissed by the Director of the International Bureau. They shall be responsible for the carrying out of the programme drawn up by the Commission. Their term of office shall not exceed four years, but they shall be re-eligible.

The budget shall determine the sums allocated to each of these classes.

6. The Director of the International Bureau shall publish the results of the work undertaken in accordance with the decisions of the Commission. He shall present to the President an annual report covering the whole field of activity of the Bureau, together with the programme of work for the following year. This report and programme shall be published.

In his capacity of General Secretary, he shall, in consultation with the President, conduct the correspondence and administer the funds. He shall be responsible for the custody of documents.

7. The Director of the International Bureau shall prepare the estimate for the budget, which, after having been approved by the President and the Commission, shall be submitted to the International Astronomical Union.

8. The income of the Commission shall be devoted to:

(*a*) Covering the expenses of administration and publication.

(*b*) Honorarium to the General Secretary as Director of the International Time Bureau.

(*c*) Grants or remunerations for calculations, observations, or experiments undertaken at the request of the Commission.

(*d*) The purchase and upkeep of material for the International Time Bureau.

The Director of the Bureau, under the control of the International Astronomical Union, shall be responsible for the expenditure allocated to the above objects.

9. The above regulations shall remain in force until December 31, 1931.

INTERNATIONAL RESEARCH COUNCIL

INTERNATIONAL ASTRONOMICAL UNION

International Central Bureau for Astronomical Telegrams

REGULATIONS

1. The purpose of the Central Bureau is to receive, centralize, and dispatch information concerning astronomical discoveries, observations, and calculations, either by telegram or by post to the various institutions or private persons subscribing to it.

2. The domicile of the Bureau shall be located at the Royal Observatory of Belgium, at Uccle; the Chief of this Institution shall act as its Director.

3. The Bureau shall be placed under the control of the Standing Committee on Astronomical Telegrams.

4. Observatories and similar institutions may subscribe at the Central Bureau for Astronomical Telegrams. The general expenses of the Central Bureau shall be divided equally between the subscribers. At the beginning of each year the Standing Committee on the proposal of the Director, shall fix the annual subscription, which shall not exceed 25 francs (French currency), payable in advance. These funds shall be used exclusively for the expenses of printing the circulars and the correspondence. In addition to his subscription each subscriber shall cover the expenses of all telegrams addressed to him.

5. A statement of all expenses shall be made at the end of each year.

6. Subscribers shall give priority to the Central Bureau in respect of information on their discoveries, observations, or important calculations. Expenses incurred in the dispatch of telegrams for this purpose shall be refunded to the senders.

7. The Central Bureau shall take the necessary measures to reach its subscribers in all parts of the world. If necessary, special agreements shall be made for this purpose with observatories or secondary centers situated in other continents which may serve as relays.

8. The Central Bureau shall make every effort to reduce the expenses of transmission to a minimum; it shall continue to employ, for this purpose, the numeric code that has been in use for several years.

NOTES

A. Telegrams sent to the Central Bureau are to be addressed:

ASTRA BRUXELLES

B. Subscribers are requested to inform the Director of the Royal Observatory of Belgium, Uccle, of the SHORTEST TELEGRAPHIC ADDRESS to which messages conveying notices of discoveries should be sent in order to reach them.

REPORTS OF COMMITTEES OF THE AMERICAN SECTION OF THE INTERNATIONAL ASTRONOMICAL UNION

REPORT OF THE COMMITTEE ON NOTATION, UNITS, AND ECONOMY OF PUBLICATION

The Committee on Notation, Units, and Economy of Publication has given careful consideration to these subjects, and, although it has no recommendations it wishes to urge upon the Union, it begs to submit the following remarks in support of the appointment of a similar international committee.

I. UNITS

The multiplicity of units, applying to the same kind of measurement, that are now used in astronomical literature, and the indefiniteness that attaches to some of them, suggest the need of a careful study of this subject and international agreement as to what units should, as a rule, be adopted.

Astronomical distances, for instance, are already expressed in terms of the kilometer, mile, "astronomical unit" (mean solar distance), light-year, "parsec" (distance corresponding to a parallax of 1″), siriometer (corresponding to a parallax of 0.″2), "the unit" (corresponding to a parallax of 0.″1), megaparsec = 10^6 parsecs, and, perhaps, a few other units.

Although no other astronomical quantity is measured in so many different units as the stellar distance there are some whose numerical values are more confusing because the name of the unit employed has at least a double meaning. Thus the "absolute magnitude" of a star may refer to its brightness in terms of some supposed constant; or to that brightness which the star under discussion would show at a distance of 10 parsecs. Similarly, the unit of heat commonly used in certain astronomical discussions, the calorie, often leaves one in doubt as to which calorie is meant, the small or the large.

Obviously, then, there is need for general agreement in respect to the units to be used.

The following suggestions are offered for discussion:

1. *Distance.*—So far as practicable let the unit distance be, *a*, the kilometer (in reference to dimensions of sun, planets, etc.). It can even be used in expressing stellar distances without employing larger numbers than the chemist and the physicist are accustomed to in speaking of the number of molecules in a cubic centimeter of gas), *b*, the "astronomical unit" (in reference to the solar system chiefly), *c*, the light-year (especially in popular articles), and *d*, the parsec (wherever appropriate), or preferably a unit 10 times as great, to be given a separate name.

2. *Absolute magnitude.*—The meaning given to the expression "absolute magnitude" should be the magnitude at the distance of 10 parsecs, at which the star's parallax would be 0.″10.

3. *Unit of heat.*—Use the joule for the unit of heat instead of calorie, as the latter is not definite unless explained.

4. *Unit of wave-length.*—Use the international angstrom.

5. *Unit of pressure.*—One bar = 10^6 dynes per square centimeter. This is in keeping with very general use in meteorology.

II. NOTATION

It is only right that considerable latitude should be allowed the author in the choice of notation, nevertheless some of the existing confusion could and should be avoided. There is no uniformity, for instance, in the notation for the elements of a visual binary system. Also different methods are used in designating the brighter stars—by constellations and Greek letters, and by right ascension and declination. Again, the notation used in celestial mechanics, though difficult to standardize, appears to be more confused than necessary.

Suggestions

1. *Binary systems.*—Use a common notation—Aitken's, perhaps.

2. *System of units.*—If metric units are used do not add the English equivalents; if English units are used add, generally, the metric equivalents.

3. *Wave-length system.*—Use the abbreviations *I S* and *R S*, and not *I A* and *R A*. Do not use the diacritical mark over either the abbreviation *A* or the word angstrom.

4. *Unit of acceleration.*—A short name is needed for the C. G. S. unit of acceleration.

Additional suggestions in reference to notation will be made in connection with "economy of publication."

III. ECONOMY OF PUBLICATION

That there is need for economy in amount of publication, in cost of publication, and in manner of publication so as to save the time and effort

of the reader, are all too evident to admit of question. The problem is how can these results be best attained.

Suggestions

1. *Observatory bulletins.*—There should be as few bulletins and as little published in them as circumstances will permit. It is better to publish in standard journals and purchase reprints.

2. *References.*—A nearer approach to standardization of references to journals, etc., than now exists should be adopted—librarian's method perhaps.

3. *Standard kinds of work.*—So far as practicable all results obtained in any one standard line of work should be presented in the same form.

4. *Summaries.*—Most articles should end with concise and accurate summaries.

5. *Catalogues.*—Catalogues of scientific literature should give concise abstracts.

6. *Economy in typesetting.*—Manuscript should be kept as free as practicable from all letters, symbols, and combinations that worry the printer or annoy the reader. Greek letters, for instance, are expensive luxuries —a dime per letter being the estimated added cost of publication. Similarly, subscripts, superscripts, inferior bars, superior bars, vertical fractions, etc., should be avoided as far as practicable.

The following recent suggestions by the London Mathematical Society are excellent:

Instead of	Always print
$\sqrt{2}$, $\frac{1}{\sqrt{2}}$, $\sqrt{13}$	$\sqrt{2}$ or $2^{\frac{1}{2}}$, $1/\sqrt{2}$ or $2^{-\frac{1}{2}}$, $\sqrt{13}$ or $13^{\frac{1}{2}}$
$\sqrt{ax^2 + 2bx + c}$	$\sqrt{(ax^2 + 2bx + c)}$ or $(ax^2 + 2bx + c)^{\frac{1}{2}}$
$\sqrt{\frac{a}{b}}$	$\sqrt{(a/b)}$ or $(a/b)^{\frac{1}{2}}$
$\sqrt{-1}$	$\imath$ or i
$n.\overline{n+1}.\overline{n+2}$	$n(n+1)(n+2)$
$(\underline{n})^2$, $\underline{n+1}$, $\underline{2n}$, $2^n\underline{n}$	$(n!)^2$, $(n+1)!$, $(2n)!$, $2^n.n!$
$\dot{x}$, $\dot{y}$, $\dot{r}$, $\ddot{\theta}$	x', y', r', θ'' (by preference)
$\frac{a}{2}$, $\frac{a+b}{3}$, $\frac{a+b+c}{4}$	$\frac{1}{2}a$, $\frac{1}{3}(a+b)$, $\frac{1}{4}(a+b+c)$
$\frac{a+b}{c}$, $\frac{a}{b+c}$, $b' + c$	$(a+b)/c$, $a/(b+c)$, $a/b + c$

$\frac{p}{q}, \frac{p}{q} + \frac{r}{s}$	$p/q,\ p/q + r/s$ (in current text)
$\frac{p}{q + r/s}$ or $p/\left(q + \frac{r}{s}\right)$	$p/(q + r/s)$
$\frac{1}{x}, \frac{1}{x^n}$	$1/x$ or x^{-1}, $1/x^n$ or x^{-n}
$\frac{p + \frac{q}{2}}{\frac{r}{3} + \frac{s}{4}}, \frac{x}{y + \frac{z}{t}}$	$\frac{p + \frac{1}{2}q}{\frac{1}{3}r + \frac{1}{4}s}, \frac{x}{y + z/t}$
$e^{-\frac{n\pi x}{l}}, e^{-\frac{x^2}{4kt}}$	$e^{-n\pi x/l}, e^{-x^2/4kt}$
$\frac{I\pi}{a}, \int_0^{\frac{\pi}{2}}, \int_0^{\frac{\pi}{n}}$	$I\pi/a, \int_0^{\frac{1}{2}\pi}, \int_0^{\pi/n}$
$(2/l)\int_0^l \sin(r\pi x/l)\sin(s\pi x/l)dx$	$\frac{2}{l}\int_0^l \sin\frac{r\pi x}{l}\sin\frac{s\pi x}{l}\,dx$*

A few of the above expressions might, perhaps, be written in still better forms. Furthermore, it is quite possible to write an integral with limits in a more condensed and more convenient form than it usually has. Thus

Instead of	Write
$\int_b^a x^2 dx$	$\int [a,b]x^2 dx$

The first, or usual, form occupies, vertically, the space of two lines. The proposed form occupies but one line.

W. J. HUMPHREYS, *Chairman*, EDWIN B. FROST, A. O. LEUSCHNER.

REPORT OF THE COMMITTEE ON ALMANACS

The Committee on Almanacs recommends to the American Section that its delegates to the Brussels meeting of July 18, 1919, be instructed to endeavor to secure the adoption by the International Astronomical Union, of the Report of the Committee on Beginning the Day at Midnight adopted by the American Astronomical Society in August 1918.

W. S. EICHELBERGER, *Chairman*, E. W. BROWN, R. H. TUCKER.

REPORT OF THE COMMITTEE ON MERIDIAN ASTRONOMY

The experience gained through the application of photography to the determining of stellar positions demonstrates that it is advantageous

* It may be noted that an integral with limits always involves a double line; so that the ordinary notation for fractions involves no additional use of "spaces" beyond those required for the integral in any case.

to secure star positions, as far as possible, by photographic methods. This leads to the suggestion of the following meridian program, which is designed to provide the necessary foundation for photographic astrometry.

The suggested plan, in brief, is to adopt 3000 fundamental stars as the initial basis of the system, to build about 40,000 reference stars upon this and then by photographic means to continue a list of reference stars to fainter magnitudes. The details of the plan follow.

I. STANDARD STARS

It is suggested that the list of fundamental stars compiled in 1910 by the Committee on Fundamental Stars of the Astrographic Chart, and published in the supplement to the Connaissance des Temps for 1914, be adopted.

II. PHOTOGRAPHIC REFERENCE STARS

As a secondary step toward providing a suitable structure upon which to base photographic plate determinations of stellar positions it is suggested that a list of stars be constructed, approximately one in each square degree, and included as far as possible between visual magnitudes 8.0 and 9.0. The list should favor the stars of the intermediary and reference lists of the Astrographic Chart, but in any event stars which have been previously observed should be selected.

III. DIFFERENTIAL MERIDIAN OBSERVATIONS

It is recommended that differential observations be undertaken in zones 20° wide, four observations being taken of each star, two in each position of the clamp. All the fundamental stars in each zone should be observed, together with a sufficient number of fundamental stars at varying declinations to link the zone with the fundamental system.

IV. REVISION OF STANDARD SYSTEMS

Because of the rapidity with which the weight of a star position diminishes, it is recommended that standard systems be revised every fifteen to twenty years, and that the results be incorporated in the ephemerides as they become available.

V. QUESTIONNAIRE

In order to disseminate information in compact form among the meridian observers, it is recommended that 6 months before each meeting of the International Astronomical Union the following questionnaire be sent to each observatory and that the reports be collated and published as part of the proceedings of the meeting.

1. Brief description of meridian work in hand, and estimated time of conclusion.
2. Instruments, character, size, etc.
3. Equipment:

(*a*) Collimators
(*b*) Mires
(*c*) Nadirs
(*d*) Impersonal right-ascension micrometers
(*e*) Personal equation apparatus
(*f*) Reversing prism on ocular—on microscopes
(*g*) Screens or other devices for magnitude
(*h*) Flexure—how determined
(*i*) Clock—how safeguarded from temperature and barometric changes
(*j*) Thermometer installation—use of whirling thermometers—Assman thermometers —or other devices
(*k*) Printing chronograph
(*l*) Devices for stabilizing mercury reflecting surfaces
(*m*) Methods of determining corrections for irregularities in the forms of the pivots

4. Remarks on any novel devices in use, with comments on the value of the same.
5. Reflected observations of stars. Are they made—how used?
6. Application of photography to meridian work.
7. Method of determining graduation errors.

VI. PUBLICATIONS

1. The present practice varies from publication of the minute detail of observations to that of giving only the final results. It is suggested that this matter be considered and that a recommendation be made in order on the one hand to save the great waste of material and effort involved in unnecessarily extended publication and on the other hand to secure the publication of as much detail as the consensus of opinion shall decide is actually desirable.

2. Uniformity should be observed in star names, in number of decimals for right-ascension and declination, and in use of declinations instead of north polar distances.

3. Standard equinoxes should be adopted.

4. No proper motions should be applied in observational catalogues to reduce from the epoch of observation to the date of equinox of the catalogue.

VII. SOUTHERN OBSERVATORIES

Attention is called to the need of more southern observatories for meridian work in view of the relative weakness of the positions of the southern stars.

BENJAMIN BOSS, *Chairman*, FRANK SCHLESINGER, FRANK B. LITTELL

REPORT OF THE COMMITTEE ON COÖPERATIVE WORK WITH THE SPECTROHELIOGRAPH

In several reports presented to the International Union for Coöperation in Solar Research, the advantages of photographing the sun frequently with the spectroheliograph at several stations widely separated in longitude have been pointed out. Without enumerating these, it is evident that a more nearly continuous record of the rapidly changing phenomena of the sun's atmosphere than we now possess would provide material for

the study of many questions, such as the possible connection between solar eruptions and terrestrial magnetic storms.

Most of the telescopes hitherto used for this work give a very small solar image, while the spectroheliographs employed with them are in very few cases of sufficient dispersion to permit satisfactory photographs to be taken with the *Ha* line. As the hydrogen flocculi, for most purposes, are of much greater interest and importance than the calcium flocculi, the object of this report is to indicate the general nature of the instrumental equipment needed to record them in such a way as to render the resulting photographs fairly comparable.

The chief instrumental requirements are:

1. A telescope giving a large solar image, not less than 10 cm. and preferably from 15 to 20 cm. in diameter. This large scale is needed in order that the minute structure defining the fields of force of the hydrogen flocculi may be clearly recorded.

2. A spectroheliograph of sufficient dispersion to permit the hydrogen flocculi to be photographed with light confined to the central part of the *Ha* line. A linear dispersion of 1 mm. = 3.5 *A* will serve very well.

It is desirable, but not essential, that this spectroheliograph should be of sufficient aperture to permit the entire solar image to be photographed at once. If this is not feasible, a second objective, giving a smaller solar image (at least 5 cm. in diameter), should be so attached to the telescope that it can be readily substituted for the objective of greater focal length.

3. It is important that spectroheliograms intended for comparative studies of eruptive phenomena and changes in the structure of the hydrogen flocculi should not differ greatly, either in the size of the solar image or in the dispersion of the spectroheliographs with which they are taken. For this reason it is desirable that the optical constants of the instruments used at coöperating observatories should be as nearly alike as possible.

4. It is desirable that the spectroheliographs should be adaptable for the photography of the calcium flocculi with K_2 and with K_3. It would also be advantageous to design them for use as spectroregistreurs des vitesses (Deslandres) and for the study of the magnetic fields in sun-spots, in case spectrographs especially designed for these purposes are not available.

A final word should be said regarding the necessity for good atmospheric conditions, to permit the sun to be photographed on the largest possible number of days and to afford the perfection of definition required to reveal the minute details of the flocculi.

GEORGE E. HALE.

REPORT OF THE COMMITTEE ON ECLIPSES

(Abstract)

The Committee believes that it might be of assistance to astronomy along the following lines:

1. To advise as to what eclipse problems are most worthy of attack, and what equipment is needed for successful work.

2. For the eclipse of 1923, to collect well in advance all available information regarding possible eclipse sites, weather probabilities, transportation facilities, living conditions, etc. (The Mexican Meteorological Service has already been communicated with in regard to the eclipse of 1923.)

3. To publish such information and eclipse computations well in advance.

4. To advise intending observers as to location so as to secure a wide distribution of stations.

5. To gain information regarding the apparatus that is available at different observatories, and to arrange as a central clearing-house for the loan of apparatus to intending observers who have not adequate facilities.

A discussion is given of the various eclipse problems under the headings:

Photography of the corona.
What is the corona?
What causes the corona?
Motion of the corona.
Polarized light in the corona.
Brightness of the corona.
Spectrum of the corona.
Wave-lengths of coronal lines and rotation of the corona.
Flash spectrum.
Einstein effect.
Times of contact.
Other eclipse problems.
Movie camera.

S. A. MITCHELL, *Chairman*, E. E. BARNARD, H. D. CURTIS.

REPORT OF THE COMMITTEE ON STANDARDS OF WAVE-LENGTH

1. It is recommended that the International Astronomical Union appoint at the earliest practicable date, a committee whose functions shall be essentially the same as those of the Committee on Wave-Lengths of the International Union for Coöperation in Solar Research.

2. *The primary standard.*—The remarkable sharpness of the red Cadmium line, the extraordinary skill with which the meter was evaluated in terms of this wave-length, and the final definition of the angstrom, as an arbitrary unit, by the international Solar Union at Paris in 1907, would seem to place the primary standard almost beyond the range of question.

However, the recent perfection of a new method for making end-standards of length, the comparative ease with which the optical length of these end-standards can be determined, and the possible superiority of some other

line (such as the bright yellow Neon line) over one from solid Cadmium as a source of light combine to urge a reconsideration even of the primary standard.

Since a committee, consisting of Messrs. Ames, Anderson, Michelson and Stratton, has been appointed by the National Research Council to consider the practicability of adopting a wave-length of light as the fundamental standard of length, it is recommended that the question of a primary standard of wave-length be held in abeyance—that is, not reopened—until the above-mentioned committee makes its report.

In case the International Bureau of Weights and Measures adopts a certain wave-length of light as their standard of length, it would appear to follow, as a matter of course, that this wave-length should be adopted also by the Astronomical Union, and that the numerical value attached to it should be that assigned by the International Bureau of Weights and Measures.

As to the possible use of a Neon line, or the sharp component of the green Mercury line, or of a sharp line in the absorption spectrum of Iodine, for the fundamental unit of length, instead of the red Cadmium line, your committee feels that this is a matter belonging entirely to the Committee of the International Bureau of Weights and Measures.

3. *Secondary standards.*—A series of secondary standards is now being determined at the Bureau of Standards and at Mt. Wilson. The Committee recommends that the Astronomical Union encourage a third series of independent measures on these lines.

It is recommended, therefore, that the International Astronomical Union arrange, if possible, with European spectroscopists for a determination of these secondary standards in Iron, Neon, and other substances which can be worked under definite and easily reproducible conditions.

The dearth of Secondary Standards in the yellow-green region and in the infra-red appears to your committee an unsolved problem. Its solution will be the discovery of a workable and easily duplicated source of light which will furnish properly distributed lines of sufficient sharpness and intensity.

4. *Tertiary standards.*—Here the greatest need appears to be for lines to fill gaps in the ultra-violet, the yellow-green, and infra-red of Iron. With reference to this need your committee has no recommendation to make except to seek further information as to how the need may be met.

For the measurement of tertiary standards independent determinations by the interferometer and by the grating are urgently recommended, on the ground that the errors of these two instruments are largely complementary. The importance of this work is emphasized by the fact that it will be scarcely practicable to print a new "Table of Stronger Lines" for the identification of impurities and similar purposes until the tertiary standards are more complete than at present.

5. *Sources.*—In order to obtain lines of constant wave-length and constant intensity-distribution, it is recommended that the modification of the Pfund arc employed at Mt. Wilson be adopted, namely, the Pfund arc operated between 110 and 250 volts as convenient, with 5 amperes or less, at a length of 12 millimeters, and used over a central zone, at right angles to the axis of the arc, not to exceed $1^1/_4$ millimeters in width.

6. *Classification.*—It is recommended that tables of wave-lengths be accompanied by at least two descriptive columns, one indicating the inherent nature of the line (intensity, sharpness, asymmetry, etc.), the other indicating the character of the measures on the line. There is urgent need of some quantitative description of relative intensity as a function of wave-length.

For indicating the intrinsic character of the line, the modification of Gale and Adams' groups, shortly to be defined at Mt. Wilson, is recommended.

For indicating the degree of accuracy, the notation of Burns (*Lick Obs. Bulletin* No. 247, p. 29 and *Bulletin Bureau of Standards*, No. 274, p. 251) is recommended.

7. *Tables of wave-lengths.*—The time for an accurate and definitive determination of the wave-lengths of the different chemical elements does not yet seem to have arrived.

It is recommended, however, that a *Table of Stronger Lines* in which wave-lengths of all elements shall be arranged in order of magnitude be prepared at the earliest practicable date. It is believed that present needs call for a table containing approximately twice as many lines as Kayser's *Tabelle der Hauptlinien*, which includes ten thousand wave-lengths and occupies one hundred octavo pages.

8. *Survey.*—It is recommended that the conference at Brussels consider the possibility and advisability of making a survey of facilities, equipment, and opportunities for spectroscopic work in the various laboratories of the world, looking toward the avoidance of unnecessary duplication and the encouragement of coöperation in research.

HENRY CREW, *Chairman*, HAROLD D. BABCOCK, KELVIN BURNS, W. W. CAMPBELL, CHARLES E. ST. JOHN.

REPORT OF THE COMMITTEE ON THE DETERMINATION OF SOLAR ROTATION BY THE DISPLACEMENT OF LINES

Since the Bonn meeting of the International Union for Coöperation in Solar Research in 1913, results of investigations of the sun's rotation have been published by a number of observers, namely, J. S. Plaskett, Hubrecht, Schlesinger, DeLury, Evershed and Royds, H. H. Plaskett, and St. John and Ware.

As the data relative to the linear velocity at the solar equator now cover

a period of some twenty years, a review of the results at this time is of interest:

LINEAR VELOCITY OF SOLAR ROTATION AT THE EQUATOR

OBSERVER	VELOCITY	NO. OF LINES	REGION	DATE
Duner	2.08	2	6301–6302	1900–5
Halm	2.04	2	6301–6302	1904
Adams	2.06	20	4196–4294	1907
Adams	2.05	22	4196–4291	1908.5
Storey and Wilson	2.08	10	6280–6318	1909
Plaskett, J. S.	2.01	19	5506–5688	1911
Plaskett, J. S.	2.02	15	4196–4291	1911
DeLury	1.97	19	5506–5688	1911
Hubrecht	1.86	40	4299–4400	1911
Plaskett, J. S.	2.01	27	4250–5600	1911–12–13
Schlesinger	2.00	20	4058–4276	1912
Evershed and Royds	1.95		3906–5624	1913
Plaskett, H. H.	1.98	12	5574–5628	1913
St. John and Ware	1.94	35	4123–4338	1914
Plaskett, H. H.	1.95	5	5900	1915
St. John and Ware	1.94	26	5018–5316	1914–18
St. John and Ware	1.95	7	6265–6337	1916–17

In view of the accuracy attainable in spectro-photographic measures the differences between these results are surprising and disappointing and are the outstanding feature in the problem of solar rotation. They are larger than the probable errors of measurement, and either depend upon systematic errors of the measurers or they are of solar or terrestrial origin. The tracing of them to their source is the line along which investigation may be directed with great advantage.

Investigations now in progress at Mount Wilson, in which simultaneous observations are made upon the two limbs and the center of the sun, indicate that temporary and local conditions in the sun's reversing layer are frequently such as to produce differences of ten per cent in the rotation values obtained by comparing east and west limbs directly.

The observations show further that high values at one limb are not correlated with high values at the other as they would be if they were due to real changes in the rotation of the reversing layer or to conditions in the terrestrial atmosphere. In the light of these results it is clear that a short series of observations may give misleading results and it even becomes a question whether the solar rotation can be determined as definitely as has been thought. The means of extended series of observations taken under constant conditions should, however, give results of great value. Such a series at Mount Wilson Observatory is now in its sixth year. The observations show no definite evidence of periodicity in the sun's rotation from 1914 to 1919, but furnish strong indications of fre-

quent local disturbances of a definite type in the sun's reversing layer, a subject still under investigation.

The formulae of Faye represent satisfactorily the relation of velocity to latitude as given by the observations of Duner, Adams, Plaskett and DeLury, and Schlesinger, but observations in the higher latitudes should be greatly extended both in number and in range of latitude. The failure of the Faye formula to represent Hubrecht's observations is probably due to the very limited time covered by them.

The only published results relating to a difference in the rotation law in the two hemispheres are those of Hubrecht. His earlier paper (1912) referred to in the report at the Bonn meeting showed an excess of the velocities in the southern over the northern hemispheres, but in the later publication (1915) covering the same epoch he comes to the opposite result basing his conclusions upon Tunstall's measurement of his plates to the exclusion of his own. The question of a difference between the two hemispheres is one of great interest and need not wait for solution until all discrepancies between observers are settled as it is a relative matter for any one observer and instrument. It, therefore, offers an inviting field of investigation at the present time, and is on the working program at Mount Wilson.

The different rates of rotation for different levels in the solar atmosphere shown by the Mount Wilson observations of Adams appear again in preliminary reductions of the present Mount Wilson series which show relatively high velocities for the strong magnesium triplet in the green, for 4227 and the H and K lines of calcium, but low values for the lines of lanthanum and of the nitrogen (cyanogen) band at 3883. The observations of Schlesinger published since the Bonn meeting agree with those of Plaskett and DeLury, Storey and Wilson, and Hubrecht in showing no positive evidence of differences in rotational velocity. It is to be noted that in the Mount Wilson observations lines differing widely in intensity, representing great differences in level, have been employed. As differences in rotational velocity must be directly related to differences in level, such lines, notwithstanding the difficulties of measurement, offer a valuable means of detecting it. The great majority of lines of medium intensity of the common elements originate in not greatly different levels, and for such lines differences in rotation and velocity would be small and correspondingly difficult to detect.

Conditions in the earth's atmosphere have been suggested by DeLury as possible causes of divergent results in observations for solar rotation. He finds evidence of the effect of an over-lapping sky spectrum in some of his observations at Ottawa, and attributed to it the large variations in rotational velocity at different times and all differences found for different elements and lines.

That a large proportion of superposed skylight might appreciably re-

duce line displacements at the sun's limb has been taken into consideration by all solar observers. Tests on Mount Wilson show that, under ordinary observing conditions, the integrated light just outside the solar image is not of sufficient intensity to affect the photographic plate during the exposure time employed for solar rotation plates; that it requires considerable cloudiness over and around the sun, conditions never approximated in regular work, to cause a measureable decrease in the line displacements at the limb; and that this large proportion of superposed skylight produces no differential effects between strong and weak lines.

Recommendations for Program of Work

1. In order to trace to their source the systematic differences that are found in the value of the solar rotation by different observers, it is desirable that further study be given to possible sources of systematic errors and to determining the source of the differences found by the same observer upon plates taken at frequent intervals, differences of the same order of magnitude as those between different observers.

2. In the meantime it is desirable to carry on, when possible, continuous series of observations under constant instrumental conditions and by the same observer.

3. As it seems increasingly probable that local conditions in the reversing layer are frequent causes of divergent results, it is advisable to extend observations over a considerable period of time in investigation of such questions as variation with latitude or differences between the hemispheres.

4. Simultaneous observation on the center and the limb is recommended as a valuable means of checking results and of investigating the hemispheres separately.

We recommend the appointment of an International Committee on Solar Rotation determined by spectrographic methods, the four points above to be referred as suggestions to this committee.

C. E. ST. JOHN, *Chairman*, W. S. ADAMS, FRANK SCHLESINGER.

REPORT OF THE COMMITTEE ON WIRELESS DETERMINATION OF LONGITUDE

The Committee recommends:

1. That the American delegation to the Brussels meeting of the International Astronomical Union urge the desirability of a Division of the Union which shall promote in the most practicable manner a world-wide program of accurate radio determinations of differences of longitudes, on the basis of one general program to be supported by the inter-allied nations concerned.

2. That the American members of this Division or Committee of the Union should be selected by the American Delegation with reference

to their fitness for planning, directing, and carrying on the American Section of the inter-allied program.

3. That the selected primary stations be so distributed throughout both the southern and northern hemispheres as to cover, as uniformly as possible, the entire world, and to give suitable checks on the work.

4. That observatories and other related institutions, both government and private, be encouraged to participate by receiving the fundamental radio signals and making simultaneous time observations, so as to link up as many positions as possible with the primary stations.

5. That the governments controlling radio stations likely to be of use be asked to give their consent to and promises of assistance in the work.

6. That Greenwich, England, be so included in the plans that the differences of longitude between it and the primary stations will be accurately determined.

7. That the determination of the difference of longitude between any pair or pairs of primary stations be made by parties from one country which would select their instruments and work out and be responsible for the result; but that nothing in this recommendation would prevent a duplication of this work by observers from another country, and, where other checks were not provided, that such duplication be encouraged.

8. That in view of the many arrangements and preparations required to secure accurate and complete results, the earliest date practicable for an international determination of differences of longitudes is the winter of 1921–22.

J. A. HOOGEWERFF, *Chairman*, W. W. CAMPBELL.

Because of his absence, Mr. J. J. Carty did not see the report.

REPORT OF THE COMMITTEE ON VARIATION OF LATITUDE

The Committee on Variation of Latitude of the American Section of the proposed International Astronomical Union submits the following report, based upon the action taken at a joint meeting of this committee with a similar committee representing the American Section of the proposed International Geophysical Union held in Washington on April 29, 1919. There were present at the meeting Messrs. Bowie and Marvin representing the Geophysical Union and Messrs. Schlesinger, Leuschner, and Littell representing the Astronomical Union. Mr. Bowie acted as chairman and Mr. Littell as secretary of the meeting.

RECOMMENDATIONS

The following recommendations are made by the committee:

1. That the observations for variation of latitude, their reduction, and their publication be entrusted to the International Astronomical Union.

2. That the continuity of the observations by the present methods at the four international variation of latitude stations, Ukiah, California; Carloforte, Italy; Mizusawa, Japan; and Charjui, Turkestan be maintained, as far as practicable, and that the matter of utilizing the observations made at other stations be considered.

3. That the question of renewing variation of latitude observations in the southern hemisphere be considered.

4. That, inasmuch as no definite plan can be advanced for defraying the expense of the variation of latitude work until some international arrangement is formulated, the several nations maintain the stations within their domains, and that some provisional arrangement be made for caring for the records, reductions, and publications, pending a permanent organization of the work.

FRANK B. LITTELL, *Chairman*, A. O. LEUSCHNER, FRANK SCHLESINGER

REPORT OF THE COMMITTEE ON ASTEROIDS AND COMETS

Among the most important objects to be served by the discovery, observations, and by researches in general on comets and asteroids, are the following:

1. Testing of gravitation by comparison of theory and observation leading to deviation from the law (e.g., those shown by the moon).

2. Testing of gravitational theories as sufficient to represent their motions for some time in the past and future.

3. Distribution with reference to theories of formation and evolution.

4. Distribution of orbits with reference to theories of the stability of their motions.

5. Positions for photometric research.

6. Photometric research for testing of theories of causes of variation of light.

7. Theories of distribution tested by search for asteroids in regions indicated by theory.

8. Positions of asteroids recorded for future researches, the direction of which is at present unknown.

These ultimate aims involve:

Observations of position, visual and photographic;

Photometric observations, visual and photographic;

Methods of observation and reduction;

Researches in orbit methods and orbit determinations, preliminary and final, and ephemerides;

Methods of special and general perturbations, of varying degree of accuracy according to special requirements;

Researches in pure celestial mechanics (e.g., periodic solutions), etc., etc.

Complete bibliographies should be prepared for each line of work.

The Committee is of the opinion that much unnecessary effort and duplication could be avoided and a thorough foundation for future researches along the lines indicated above could be furnished by first of all bringing together all published material in a bibliography which would give in abstract form the results of past observations and investigations. Observations of positions, photometric observations, etc., would be listed merely by dates of observation and references. At the present time there is completely lacking a reference work for asteroids such as Galle's Comet List. The latter also requires to be brought up to date; several fragmentary continuations of Galle's list are in existence. Bauschinger's Statistik der Kleinen Planeten gives merely statistical information based on adopted asteroid orbits, without giving a complete history of the work done on the individual asteroids, including references to the dates of observations.

The research requirements along the various lines indicated above cannot be fully stated until a complete history and bibliography for comets and asteroids are available. Galle's list has been of incomparable value in this direction, particularly in the identification of comets. After the history and bibliography have been completed the research requirements along the various lines indicated above should be published.

Arrangements should be made by international coöperation for the publication of opposition ephemerides.

After the plans have been outlined in the rough and published, a census should be made of observatories and astronomers, for the purpose of securing volunteers for specific work of observation or research. The results of this census should be published, so that every astronomer will know to whom to turn for information of any kind regarding asteroids, and so as to avoid the present useless duplications, particularly of observation.

The general direction of these plans should be in charge of an international committee composed of the chairmen of the Committees on Comets and Asteroids of the various national sections.

As a first step toward the realization of the foregoing plans it is necessary to adopt a distinct classification for comets and asteroids. An example for a suitable asteroid classification is attached as Appendix A. This classification is based on the present state of observation and computation and on the scientific importance of the object. It at once enables the astronomer interested in a particular kind of research to select the object and the material needed.

The present designation of asteroids might be supplemented by using the letters *a*, *b*, *c*, etc., for easy recognition of the condition of the available observational material and of its orbit theory, an asterisk being added to designate those of scientific importance. These asteroids might be given a description to indicate the nature of their scientific importance,

thus: P_1 = Trojan Group, p_2 = Hecuba Group, etc.; v for variable, l for libration, e for eccentricity, i for inclination, etc. The magnitude at unit distance might also be given. An investigator could thus pick out at a glance the asteroids of particular interest in his line of research.

After each asteroid has been properly classified and designated, cards should be completed in order of importance. This work should all be done under the auspices of the International Committee, provided such a committee adopts some plan as outlined here.

Ephemerides.—An international convention will have to be held for providing observations and preliminary orbit determinations for newly discovered planets and for carrying forward the computations. It would seem wise at this time for the international committee to canvas the different countries as to the share of burden each can undertake. At the outset probably different countries in the Union would care for the asteroids discovered in their own country, without attempting a complete duplication of the work now in progress at Berlin. In this connection, the names of the observatories and astronomers who have undertaken to secure the necessary and sufficient observations in the case of each planet and to carry forward the computation should be published.

It is very essential that an understanding be reached to secure the necessary and sufficient observations of a newly discovered object to secure a satisfactory orbit sufficient to locate the body of the next opposition and to avoid all unnecessary observations and computations.

After the International Committee has adopted definite plans in regard to the foregoing points national observatories and almanac offices will no doubt be in a position to care for such ephemerides as involve routine computation only. The whole work, however, should be organized under the auspices of the International Committee.

As there may be differences of opinion no definite plans are proposed until the different countries are able to state what assistance they can render.

It is desired that monographs be published for the benefit of various groups of research men by persons invited to undertake this task by the committee, or by volunteers.

Recommendations.—In particular, the Committee makes the following recommendations:

1. That the Executive Board of the National Research Council be requested to secure funds for the preparation of a history and bibliography of comets and asteroids, with the understanding that this work will be undertaken by the United States.

2. That the International Astronomical Union appoint an International Committee on Comets and Asteroids.

3. That the American Delegation to the International Astronomical Union submit to the International Committee the proposals of the American Committee on Comets and Asteroids for their approval.

4. That in particular a preliminary distribution of the necessary observational and computational activities among the countries represented in the Union be agreed upon, on the basis of such offers as may be received from the different countries.

5. That the American Delegation pledge that it will arrange for the care of the observational and computational work incident to comets and asteroids discovered in the United States.

A. O. LEUSCHNER, *Chairman*, E. W. BROWN, G. H. PETERS.

APPENDIX A

MEMORANDUM ON CLASSIFICATION OF PLANETS

Class a.—Asteroids for which accurate definitive orbits and general perturbations are available.

1. *Ordinary asteroids* which have no immediate scientific interest. Observations of these should be suspended, but approximate ephemerides should be provided for identification purposes, if accidentally observed.
2. *Important asteroids* of further scientific interest should be observed in accordance with prearranged coöperative plans with the object of securing sufficient observations for the scientific purposes in hand, and avoiding unnecessary multiplication of observations. Ephemerides should be published, the accuracy of the ephemerides in each case to depend on the research requirements for that case.

Class b.—Asteroids for which accurate definitive elements are available, but no general perturbations.

1. *Ordinary asteroids.* Same as *a* (1). Approximate general perturbations should be computed as a later part of an asteroid program, as opportunity presents itself, preferably only the principal terms in groups.
2. *Important asteroids.* Observations to be dispensed with until general perturbations have been computed, degree of accuracy to depend on particular scientific requirements in each case, then same as *a* (2).

Class c.—Asteroids for which satisfactory elements are not available, but which have been sufficiently observed for the determination of satisfactory osculating elements.

1. *Ordinary asteroids.* Same as *a* (1). Elements to be improved, then the same as *b* (1).

   Asteroids for which osculating elements should be derived, with the use of special perturbations, accuracy in each case to depend on the special research requirements; then same as *b* (2), then *a* (2).

Class d.—Asteroids for which sufficient observations are not available for satisfactory orbit determinations. These should be observed accord-

ing to coöperative plan to secure necessary and sufficient observations to bring them through the preceding classes into *a* (1) or *a* (2).

Class e.—Lost asteroids. These should be made the object of special orbit investigations to ascertain range of solution, and most probable orbit.

Class f.—New discoveries. A fairly definite observational programme for necessary and sufficient observations during discovery opposition should be agreed upon, the discoverer to be responsible for the execution of the observational programme in coöperation with one or two other observatories, the orbit determinations to proceed hand in hand with the observations, so that a fairly accurate orbit may be available at end of discovery opposition to serve for incorporation of asteroid in one of the preceding classes.

Accidental discoveries to be reported to central agency or agencies for assignment of observational programme and orbit computations.

Particularly interesting cases, such as 1911 MT, or object Wolf 1918, etc., to be generally distributed by telegraph to stimulate observation and investigation.

Class g.—Asteroids observed insufficiently even for preliminary orbit determinations.

SUPPLEMENTARY RECOMMENDATIONS OF COMMITTEE ON COMETS AND ASTEROIDS

Recommended for Reference to International Committee on Comets and Asteroids

1. That mean places of asteroids and comets visually observed with an equatorial be published instead of apparent places. At present the orbit computer makes no essential use of the apparent place reduction of the comparison star. It is practically universal to employ the mean place, corrected or uncorrected for annual aberration according to method, obtained by forming apparent place of object minus apparent place reduction of comparison star. This mean place may also be obtained by applying $\Delta\alpha$ and $\Delta\delta$ directly to mean place of comparison star. In case the orbit computer wishes to take account of the differential apparent place reduction, he would, if apparent place were discontinued, be limited to the use of the differential formulae, which, however, are not longer than a single apparent place reduction. Thus it is seen that the proposed change would give the orbit computer directly what he uses; would in no way increase his labor; would save the observer an apparent place reduction for every observation; and visual observations would be presented in the same way that photographic now are.

2. That each observatory adopt its own distinctive provisional designation for asteroids not identified up to the moment of publication with a previously discovered one. Washington has chosen *W* and Johannesburg

has chosen *T*, followed by a number, but it might be anything distinctive. It is advisable that this always be preceded with the year of discovery, thus, 1917 *W*15, 1912 *T*16, etc. This will enable an asteroid to be referred to by a *distinctive* designation in current discussion and in indexes, thus facilitating such discussion and the looking up of references.

3. That this provisional designation of asteroids should be retained until undoubted identification with an old asteroid or until identified at a second opposition, when it may be given the permanent number designation. This would prevent holes in the consecutive numbering such as (132) Aethra, etc.

4. That the term "astrographic," as first used in Greenwich Observations 1910, be more generally adopted to denote the position of an object obtained from a photographic plate, by using in the reduction the mean places of comparison stars at the beginning of the year. Neglecting the differential apparent place reduction, it is identical with the position obtained by applying $\Delta\alpha$ and $\Delta\delta$ directly to the mean place of the comparison star, as mentioned in (1).

5. That observers be urged to publish observation times in decimal of day, G. M. T. (Greenwich Civil Time after 1925, if the present agitation results in discontinuance of astronomical mean time). For this G. M. T. decimal of day is the form which most computers use, since it is most convenient, because the national ephemerides use G. M. T.

6. That the ephemerides give the rectangular solar coördinates at the beginning of the year, since they are most made use of by orbit computers who almost universally use mean place at beginning of year, and also to give these coördinates in the usual manner so that they may be readily differenced. The Berliner Jahrbuch has done this since 1868. The reduction to some other epoch such as 1900 or 1925 would also be useful. In addition to the coördinates the velocities of the coördinates per one-tenth of a day should also be given to facilitate interpolation and orbit computation by Laplacean methods.

7. That 1900.0 be adopted for the publication of positions of asteroids and for orbit computation. Many times more reference stars for both photographic and visual purposes will be referred to 1900.0 when the astrographic program is completed than to any other equinox. Perhaps more photographic positions of asteroids are now published than visual and probably in a few years they will greatly outnumber the visual. Since it is easier for the photographic observer to derive positions in the equinox of the catalog, it would seem that in the long run, more computation will be saved by this coöperation of observer and computer than by the adoption of any other equinox or equinoxes.

A. O. LEUSCHNER, *Chairman*, E. W. BROWN, G. H. PETERS.

REPORT OF THE COMMITTEE ON STELLAR PARALLAXES

At the present time determinations of stellar parallax by the trigonometric method form part of the regular work at the following observatories:

	APERTURE	CHARACTER	FOCAL LENGTH
Allegheny	30 inches	Photographic Refractor	46 ft.
Dearborn	18.5	Visual Refractor	28
Greenwich	26	Photographic Refractor	23
McCormick	26	Visual Refractor	32
Mount Wilson	60	Reflector	80
Swarthmore	24	Visual Refractor	36
Yerkes	40	Visual Refractor	62

At these seven institutions material sufficient for the determination of three hundred parallaxes is obtained each year. The average probable error of one determination is under .01″.

Determination of stellar distances by the spectroscopic method are being carried out at Mount Wilson with the 60-inch reflector. When the 100-inch telescope is in regular operation, a larger part of the time of the 60-inch telescope will probably be devoted to obtaining spectrograms for this purpose, so that we may expect a rapid increase in our knowledge from this source. This committee recommends that other institutions be urged to undertake this work, their programs to include a large number of objects already observed by the spectroscopic method. Attention is called to the fact that existing spectrograms (obtained, for example, in radial velocity investigations) could profitably be examined from the point of view of absolute magnitudes and stellar distances, at least for the spectral types Fo to M.

The paucity of reliable parallaxes determined by the trigonometric method has hitherto been a serious obstacle to the effective application of the spectroscopic method, since the latter depends principally upon the trigonometric results for the determination of relationships between line intensities and absolute magnitudes for the various spectral types. The war has delayed the publication of much material that will overcome this obstacle to a considerable extent, and it is likely that in a few years ample material of this kind will be at hand. One of the most important services that observers employing the trigonometric method can render at the present time is to arrange their programmes with the needs of the spectroscopic method in mind.

This committee makes the following recommendations in addition to those already mentioned:

1. That observers employing either method should record the reason for each star being placed upon their observing lists, in order that their results may be properly used in statistical discussions.

2. That they should aim to observe a large number of stars in duplicate in order that data may be forthcoming for the determination of systematic differences and systematic errors.

3. That a catalogue of stellar distances in brief form be published in the near future.

4. Most important of all, that steps be taken at once to determine stellar distances by both trigonometric and spectroscopic methods in the southern hemisphere. In no respect is our knowledge of southern stars as meagre as in this.

Finally, it is recommended that the International Astronomical Union appoint a committee to promote coöperation along the lines indicated.

FRANK SCHLESINGER, *Chairman*, W. S. ADAMS, S. A. MITCHELL.

REPORT OF THE COMMITTEE ON STELLAR PHOTOMETRY

The following observations and suggestions are submitted for the consideration of the American delegates to the meeting of the International Astronomical Union, with the hope that they may be helpful in the organization of an International Committee on Stellar Photometry. It is for this purpose that they are presented, rather than as a program for the operations of such a committee when once formed.

The organization of an International Committee on Stellar Photometry, which we herewith recommend, requires an immediate specification of the functions to be exercised. These we believe may be summarized as follows:

(1) To advise in the matter of notation, nomenclature, definitions, conventions, etc., whose universal adoption will simplify and unify the publication and use of photometric results.

(2) To plan and execute investigations requiring the coöperation of several observers or institutions. The necessity for coöperation in such investigations may arise from geographical considerations, from the magnitude of the undertaking or from the demands for precision which can be satisfied only by combining the results obtained with different instruments and under widely different conditions.

Thus the systematic observation of variable stars requires operations in both northern and southern hemispheres and in different longitudes; again, such an undertaking as the reduction of the provisional magnitudes of the *Carte du Ciel* to the normal scale, because of the labor involved if for no other reason, could not be carried out by any single institution; and, finally, the peculiar difficulties and numerous sources of error involved in the establishment of reliable standards of brightness raises an imperative demand for confirmation which can be satisfied only by concerted and coöperative effort.

The nature of the activities included under (1) is sufficiently obvious from the above formulation. There can be no question as to the desira-

bility of a central committee charged with decisions relating to notation, nomenclature, etc. For example, in the matter of naming newly discovered variable stars, endless confusion can be avoided only through the action of a centralized body of recognized authority and responsibility. Nevertheless some elaboration of statement is necessary in order to emphasize what we believe to be a sound basis of procedure.

A sharp distinction should be drawn between the adoption, on the one hand, of definitions and conventions, and, on the other, of the numerical results which flow from the introduction and usage of such definitions and conventions. The adoption of a definition or convention naturally should never be undertaken hastily or without detailed consideration of all the attendant circumstances; but once an appropriate formulation has been attained, there need be no hesitation or objection to its universal acceptance. Thus we find everywhere in use the definition of the scale of magnitudes represented by the formula

$$m - m_0 = -0.4 (\log I - \log I_0).$$

Similarly, the convention adopted by the committee on magnitudes of the *Carte du Ciel* relative to the zero point of the photographic scale of magnitudes finds ready acceptance on the part of all observers. Or again, the definition of color-index as the difference: photographic magnitude *minus* visual or photovisual magnitude, affords so convenient and useful a measure of the color of a star that it meets with the approval of everyone engaged with the problems of stellar photometry. Thus we find introduced into astronomical practice a uniformity and standardization which is only useful and helpful.

The practical application of these definitions and conventions requires, however, the usage of numerical standards and constants. And here caution must be exercised if progress is really to be aided. A definition or convention in the nature of the case possesses elements of permanence and invariability; but a numerical constant is a thing of the day, to be discarded the moment a more precise determination becomes available. The quality of inconstancy, the ever-increasing precision which characterizes the metrical results of science, should always be borne in mind; otherwise there is danger that the seal of authority may be placed upon results in which authority should have no place. The thing to be emphasized is evidence, not authority, and the evidence should be frequently reviewed.

Thus in the matter of magnitudes: it will be necessary in undertaking any coöperative investigation to specify the standards which are to be used. The selection naturally will be made by the International Committee after a critical examination of the evidence underlying the results then available; but it should be clearly understood that the standards thus chosen are for that particular investigation and are not necessarily to be used for any other which subsequently may be undertaken. In the present

state of the question it seems undesirable that any particular set of standard magnitudes be designated by the adjective "international." Such an action on the part of an International Committee would necessarily convey the impression of an authoritativeness and finality of decision which it would be difficult to justify in view of the present outstanding differences in the results of different observers.

This does not mean that there should be no international standards. On the contrary, standards to which this designation may properly be applied should be established. But this can be attained only through a coöperative effort of the type specified under (2). The question will be reverted to later. An additional remark should be made here, however, as a supplement to the above considerations, namely, that the provisional character even of any act of "international" standards should be emphasized, say, through some such designation as I. C. P. 1920 (International Committee on Photometry, Standards of 1920).

We believe, too, that much conservatism should be displayed in the matter of standardization of methods and equipment. As an illustration, it is well known that photometric results obtained by visual methods depend upon the color perception of the eye of the observer. Thus for the Harvard durchmusterung (*Harvard Annals 45*) and the durchmusterung of Müller and Kempf (Potsdam Pub. 9), both catalogues of visual magnitudes, Schwarzschild finds (Göttingen Aktinometrie B 16)

$$H - P = -0.27 - 0.04(P - 6.5) + 0.23\,(G - P)$$

in which $G - P$ is the color-index obtained by comparing Göttingen photographic with Potsdam visual magnitudes. The last term of the equation represents the differences which are to be attributed to peculiarities of color-perception. In visual photometry naturally there can be no standardization in the sense in which the term is used here. Each observer must work with the eye with which he has been provided.

Of late years, however, photovisual magnitudes have been much used as a substitute for those derived visually. An isochromatic plate exposed behind a suitably chosen yellow filter affords results which are at least approximately the same as those obtained visually. But photovisual magnitudes are obviously dependent upon the color-sensibility of the particular kind of plate and filter employed, and in this connection it is frequently suggested that a standard plate and filter be universally adopted and used for all photovisual observations.

Unfortunately the difficulties are not so easily to be avoided. The suggestion overlooks a third contributing factor whose significance is not generally appreciated, namely, the telescope. The selective absorption by the glass of the objective or by the silvered surface of the reflector plays a very important part in determining the color-sensibility of the equipment. Moreover, it is not merely a question of refractors as against

reflectors, for different objectives seldom behave exactly the same and not infrequently very large differences are to be met with. This is best illustrated by the following comparisons of different series of photographic results. Here the complicating influence of a filter does not enter, because no filter has been used, and the effect of the kind of plate is largely eliminated, because with some exceptions the same brand of plate has been used throughout. The comparisons show only the term depending on the color-index C.

MW—Greenwich	$= +0.27C$	Mt. Wilson Cont. No. 97, p. 28
MW—Dziewulski	$= -0.06C$	
MW—16-in. Metcalf	$= +0.33C$	Mt. Wilson Cont. No. 98, p. 5
MW—8-in. Draper	$= +0.06C$	
8-in. Draper—0.5 in. Ross-Zeiss	$= -0.13C$	
8-in. Draper—4-in. Cooke	$= +0.21C$	

Other examples might have been added, but those given illustrate the magnitude of the effect and show the impracticability of restricting the photographic measurement of stellar brightness to any closely specified region of the spectrum, for the observer must use the telescope which is available.

Emphasis should, therefore, be directed, not toward standardization, but toward the publication of results in such a form that they may be reduced to *any* system for purposes of comparison and combination with other results. Apparently this is to be accomplished only by giving the color of a star in addition to its brightness. Spectral type cannot be regarded as a satisfactory substitute for color, for it is now known that the color-indices of stars of the same type-color, especially the *G*'s and *K*'s—may differ by half a magnitude or more.

Admitting the impracticability of basing the observations upon any prescribed system of color (in the sense that the measurements are to be made within certain specified limits of wave-length), one might still ask if it is not desirable that the results be reduced to a standard system in advance of publication, in order that all catalogue values may be directly comparable. The answer here is that in the present state of photometry the reduction factors for color will necessarily be uncertain, and hence subject to correction. It thus appears important that the results referred to the color system of the equipment used should always be included in the published results, even when reduced values are also given.

These details are presented as an indication that reservation and caution must characterize the decisions of a central committee in case its activities are to be helpful; at the same time they reveal the disconcerting fact that measures of stellar brightness will attain their full value only when results for the colors of the individual stars become available. This is strikingly shown by an attempt to compare the results of the Durchmusterung of the Selected Areas recently issued as *Harvard Annals* 101 with photographic

magnitudes for these same regions determined at Mount Wilson. The color-sensibility of the two instruments—the 16-inch Metcalf and the 60-inch reflector—is so different that the comparison is meaningless except for the few stars whose colors are known. The differences in this instance are extreme, because of the exceptional behavior of the 16-inch Metcalf instrument. A similar situation must be met in comparing the magnitudes of the Polar Sequence determined at Greenwich with those of other investigators. The influence of the instrument is here also unusually large; nevertheless such differences always appear, to some extent at least, and, unless taken into account, vitiate the comparison of results with different instruments. This fact was recognized by Miss Leavitt in her investigation of the Polar Sequence and allowed for wherever possible. The resulting color system in *Harvard Circular* 170 is not very different from that of the Mount Wilson photographic magnitudes; but the true relation of the results of these two investigations can be determined only by allowing for the difference in the two systems. Owing to the systematic difference in the color of bright stars and faint stars, there is a direct influence upon the scales, whose accumulated effect between the 6th and 10th magnitudes amounts to nearly a tenth of a magnitude.

Before leaving these general considerations, and more or less as a summary of them, it may finally be remarked that in our opinion the most effective coöperation is to be attained by granting to the individual investigator the greatest freedom of initiative and operation consistent with the ends to be accomplished. What is desirable as an end can profitably be determined by consultation and mutual agreement; but a healthful development of research requires that the individual be responsible for the means by which its objects are to be attained.

The organization of an International Committee will require a classification of the activities which it is to undertake and the appointment of various subcommittees. These naturally will find their origin in the questions which the committee must consider, hence it seems desirable that a few of the most important of these problems should be passed in review.

1. *Standards of magnitude.*—With proper allowance for differences in color systems and zero points, the photographic scales of Harvard, Greenwich (Chapman and Melotte), Potsdam (Dziewulski), and Mount Wilson are in satisfactory agreement between the 10th and 16th magnitudes. The interval including the brighter stars is fully covered only by the results of Harvard and Mount Wilson. Here there is a scale divergence of 6%. For the reconciliation of this difference further investigations will be required, which, in part at least, should be in the hands of other observers. The question is, therefore, very definitely one for the consideration of an international committee. The observations should preferably be made on the stars of the Polar Sequence. In any event groups such as the Pleiades which contain no red stars that are bright should be avoided, owing

to the fact that the gradual increase in color with increasing magnitude makes it difficult to distinguish the influence of the color equation of the instrument from that of an error in the scale.

The visual and photovisual standards now available also require further attention. These will most conveniently be considered in connection with the photographic standards.

When once it is clear that all serious systematic errors have been eliminated, the committee may with propriety consider the combination of the results of the different investigations for the formation of series of international standards, which doubtless would then require revision only at infrequent intervals.

It would be a convenience to observers if the adopted values were referred to alternative systems of color: in the case of photographic magnitudes, for example, to the color system of an average objective, say, that of the 8-inch Draper telescope which is essentially that of the Harvard Polar Sequence, and also to the color system of the reflector, which very likely is constant or nearly so for all instruments of this type. The visual standards should be referred to the color system of the Harvard Photometry; but photovisual standards based on the reflector should also be available. The standardization of a plate and filter for use with the reflector may not prove feasible; but having chosen an isochromatic plate which is easily obtainable it doubtless would not prove too difficult to construct a filter such that the combination reproduces the adopted standards. Because of its bearing upon various physical questions such as stellar temperatures, the spectral intensity-curves corresponding to the color-system used should in all cases be published.

2. *Reduction of the magnitudes of the Carte du Ciel to the normal scale.*—This undertaking has been in the hands of the Committee on Magnitudes of the *Carte du Ciel*. In case the work of this committee is merged with that of the organization under discussion, the investigation will immediately become one of the most important and pressing questions before the newly organized committee.

Two methods of procedure are possible: (*a*) Await the results of (1) above, and then use the resulting standards; (*b*) establish the scale for the Astrographic Zones with the aid of the valuable counts recently published by Turner. This would require a knowledge of the star-density as a function of galactic latitude and longitude for each interval of magnitude. Each of these methods has its advantages and disadvantages which will require the attention of the committee.

3. *Nomenclature of newly discovered variable stars.*—Hitherto in the hands of the Astronomische Gesellschaft. An important question of policy must be faced in this connection. Probably the Argelander notation should be continued, because of its convenience. For catalogue purposes, however, the Harvard notation has important advantages, be-

cause of its elasticity. The number once assigned is permanent and always places the star in its proper serial order in the catalogue.

4. *Ephemerides of variable stars.*—Hitherto published annually in the *Vierteljahrsschrift.* Again a question of policy must be considered. In case this very desirable aid to the observer is to be prepared under the direction of the International Committee, the form of publication should be most carefully considered.

5. *Catalogue of variable stars.*—Intimately related to (3) and (4). A complete catalogue, of carefully considered form, should be published; at intervals which may be determined by experience it should be completely revised. In the interim annual supplements containing revisions and additions should be available.

6. *Systematic observation of variable stars.*—To insure a proper distribution of effort and to avoid a needless duplication of results, there should be mutual agreement between observers as to the objects to be observed or at least an announcement as to objects under observation. A central committee, as a clearing house for the dissemination of information and the arrangement of details, would prove invaluable. Directive action on the part of the committee should be reduced to a minimum. The affiliation of associations of amateurs should be invited.

Other questions might easily be added, but these we believe sufficient, to indicate the character of the organization required.

Very respectfully submitted,

F. H. SEARES, *Chairman*, S. I. BAILEY, F. C. JORDAN, J. A. PARKHURST, JOEL STEBBINS.

REPORT OF THE COMMITTEE ON DOUBLE STARS

I. CLASSIFICATION

The term "double star" is still loosely used to describe any pair of stars which have been connected by micrometric measures. It seems desirable to agree, if possible, upon some more exact definition and to adopt some system of classification and of printing conventions which will enable us to distinguish between pairs of different character. To be of service, such a system must be adopted by common consent of astronomers and such consent can best be obtained by discussion and agreement at a meeting of the character of the proposed International Astronomical Union. As a basis for discussion, the Committee on Double Stars suggests:

1. That in measuring and cataloging *new pairs* definite limits as to magnitude and angular separation be observed, such as those, for example, proposed in *The Working Definition of a Double Star*, published in *A. N.* **188**, 1911, (281).

2. That double stars already cataloged be divided into four classes, as follows:[1]

[1] The section requested the committee to devise a different mode of printing.

Class A.—Systems for which the measures show a definite curvature of the relative path, or a well-determined variation in velocity along the relative path. Class A systems are to be indicated in catalogs by the use of small capitals; e.g., 61 CYGNI, Σ 483, δ EQUULEI.

Class B.—Systems which do not come in Class A, but whose physical connection is probable, as shown by the fact that the components have the same proper motion. Since limits must be adopted to exclude stars belonging simply to a great cluster (e.g., Ursa Major) it is suggested that pairs be entered in Class B only if the angular separation is less than 100 times the annual proper motion. Pairs under 1″.0 separation which have been under observation, say, for a decade or more, and have shown no relative motion may be included in this class on the assumption that they are moving together at a rate less than ″.01 per annum. Systems of Class B are to be indicated in catalogs by the use of bold faced type; e.g., **Σ 110, OΣ 175, OΣ 424.**

Class C.—Systems whose components have from the measures apparently different proper motions. This class will include the purely optical pairs and also pairs which may later require transfer to Class A. In catalogs they are to be indicated by the use of italics; e.g., *δ Herculis*, *Σ 2760*, *Σ 742*.

Class D.—Systems about whose motions nothing is known. This class might be made to include the wide, faint pairs apparently simply of historic interest as well as new pairs of unknown proper or relative motion. It is evident that pairs entered in this class will ultimately be transferred to one of the three preceding classes or dropped entirely from our catalogs. They will be indicated in catalogs by the use of lower case Roman type; e.g., h 528, A 3000.

II. COÖPERATION

It is highly desirable that double star observers work in closer coöperation than heretofore to insure (1) the sufficient observation of all double stars in need of measurement at any given epoch; and (2) the avoidance of unnecessary duplication of measures. Of course no action should be taken that would in any way discourage personal initiative in the choice of problems or methods; but it is suggested that certain astronomers who have been specially interested in collecting all measures of double stars (e.g., Messrs. Eric Doolittle, Robert Jonckheere, R. T. A. Innes) might be asked to act as a "clearing committee," which would supply any observer wishing it with a list of double stars in need of measurement and, in principle, suited to the power of the telescope to be employed.

III. GENERAL CATALOGS

Experience in using and handling Burnham's General Catalog has proved the inconvenience of having the data for any one system recorded

partly in one volume and partly in another. It is, therefore, recommended that in future catalogs all data for a particular system be printed in one place. For catalogs printed in the near future the epoch 1950.0 is recommended for the R. A. and Decl.

The material for each pair should include, in addition to measures indicating the character of the motion, the magnitude of the star (combined brightness of the components) on some consistent scale (which might well be that adopted in the photometric magnitudes of the Henry Draper Catalogue), the spectrum (when known), and the proper motion (in *polar coördinates*, not in R. A. and Decl.). For systems having known orbits all the elements, and not simply the period, should be recorded. It is not at all essential that all measures should be recorded, but references should be given to all published measures, orbits and notes. This, essentially, is Professor Doolittle's plan for the extension of Burnham's General Catalog.

The ideal catalog would cover the entire sky. If this ideal cannot be attained, it is recommended that those who plan the publication of partial catalogs come to an agreement as to limiting declination so that unnecessary overlapping may be avoided.

IV. WORK IN THE SOUTHERN HEMISPHERE

The most important piece of observational double star work that can be undertaken at the present time is the extension to the South Pole of a survey of stars to about the 9th magnitude along lines similar to those of the survey of the northern sky completed at the Lick Observatory, and the systematic remeasurement of all the closer pairs in the southern sky which have already been cataloged. A careful examination of all published data indicates that at least 1800 double stars with angular separation under $5''$ remain to be discovered among the stars as bright as 9.0 magnitude in the southern half of the sky—two thirds of them in the area south of $-30°$ declination. This assumes the southern sky to be as rich in such systems as the northern. Further, there is only one system (α Centauri) south of $-40°$ for which we have satisfactory orbit elements; there are at least 18 such north of $+40°$.

At present the only workers in this field are Mr. Innes, at Johannesburg, using a 9-inch refracting telescope, and Mr. Dawson, at La Plata, using a 17-inch refractor. A larger telescope in a favorable position is urgently needed. Using a telescope of 20 to 25 inches aperture in a good location, a competent observer could extend to the South Pole such a survey as has been mentioned in about six years and could at the same time secure measures of all close known pairs south of $-30°$ declination. Any action that can be taken by the International Astronomical Union to encourage and assist such an undertaking either through one of the observatories already existing in the southern hemisphere or through a special expedition from the north is heartily to be recommended.

V. COMMITTEE ON DOUBLE STARS

Finally, your Committee recommends the appointment of an international committee on double stars to act under the authority of the International Astronomical Union.

ROBERT G. AITKEN, *Chairman*, ERIC DOOLITTLE, W. J. HUSSEY.

NOTE.—As the committee's work was carried on by correspondence there was not time for a complete discussion in advance of the meeting of the Section. The following comments were added by Mr. Hussey:

"1. Personally I do not attach very much importance to the formal definition of a double star. I am inclined to accept our heritage from the past and work according to our best judgment for the future. The attempts made to define a double star in terms of magnitude and distance have generally broken down by not being sufficiently inclusive, owing to the movements of the stars themselves.

"2. It does not seem to me wise to complicate the printing of results by the use of different kinds of type for the different classes of double stars, such as capitals, small capitals, italics, lower case, and bold face, etc. Such attempts lead to unsightly printed pages, and to the necessity of keeping constantly in mind the nomenclature used. In my opinion it is better to follow the prevailing practice of using brief descriptive terms, which carry their own meaning, such as binary, optical, pair, fixed, etc.

"3. The visual double stars are evidently binaries or optical pairs. To which class a given star belongs can only be ascertained from a sufficiently extended series of micrometrical measures, and such a series may require many years. Until such measures are obtained the status of a pair remains undetermined. It is not justifiable, as is proposed in the section "Class B" of the Report, to regard all pairs whose distances are under 1".00 as having common proper motions when no change is shown by measurements extending over so brief a period as ten years. Longer series of measurements may show some such stars to be optical pairs, others binaries, and still others common proper motion pairs.

"4. Pairs whose components have different proper motions are optical systems. No star of this class can be a binary, and, therefore, appropriate to be put in Class A, as suggested in the paragraph headed Class C of the report.

"5. Northern Work. At the present time the most pressing need in the north is the remeasurement of the pairs discovered during the past twenty years, and especially the close pairs discovered at the Lick Observatory.

"6. Southern Work. In the south there is needed the measurement of the known pairs, not recently measured, and the examination under good conditions of the southern stars to the ninth magnitude for the discovery of new pairs.

"In this connection it may be stated that the Observatory of the University of Michigan began some years ago the construction of a 24-inch refracting telescope for double-star work in the Southern Hemisphere. The mounting for this instrument was nearly completed five and a half years ago, when work upon it had gone as far as was practicable until the objective could be obtained. As soon as the glass for the objective can be obtained this instrument will be completed, and then taken to the Southern Hemisphere for double-star work and for other observations. No site has been definitely selected for this instrument. It will be taken to that place which promises the best results.

W. J. HUSSEY."

REPORT OF THE COMMITTEE ON STELLAR CLASSIFICATION

Your Committee recommends the following proposal for adoption by the American Section, and presentation by its delegation at the international meeting at Brussels.

CLASSIFICATION OF SPECTRA

(*a*) That the methods of determining the spectral class, according to the Draper Classification, which have been developed by Adams and Kohlschütter, be adopted as an extension of the international standard of classification.

(*b*) That the notation of the Henry Draper Catalogue (*Harvard Annals*, vol. 91) be adopted in the case of stars of Classes R and N.

(*c*) That the notation of the Henry Draper Catalogue be adopted for the spectra of gaseous nebulae, and that P50 be employed to denote spectra intermediate between gaseous nebulae and fifth-type stars.

(*d*) That a series of standard stars be chosen, representing typical spectra, separated by not more than five divisions of the Draper scale (such as B0, B5, A0, etc.) and that reproductions of negatives of these spectra be published for the benefit of astronomers generally.

(*e*) That the presence of the more noteworthy spectral peculiarities (such as bright lines, unusually sharp or diffuse lines, and the peculiarities in intensity associated with bright or faint absolute magnitude, and with the Cepheid type of variation) be denoted by distinctives ymbols appended to the ordinary designation of the spectral class.

(*f*) That a Committee on Stellar Classification be appointed by the International Astronomical Union, which shall undertake the preparation of the list and reproductions of standard spectra (recommendation (*d*)) and the formulation of the notation for spectral peculiarities (recommendation (*e*)) after consultation with the most experienced investigators in this field.

In explanation of (*e*) it may be noted that the use of new distinctive symbols for spectral peculiarities is proposed because the small Roman

letters at present employed to denote certain characteristics are not used in the same sense in different spectral classes.

The remaining recommendations explain themselves, and all the recommendations have received the unanimous approval of the members of the Committee, either during preliminary discussion, or in their final form.

H N. Russell, *Chairman*, W. S. Adams, A. J. Cannon, R. H. Curtiss.

REPORT OF THE COMMITTEE ON RADIAL VELOCITIES

We beg to recommend to the American Section of the International Astronomical Union:

1. The formation of a Committee on Radial Velocities, in the International Astronomical Union, whose duty it shall be to encourage and promote research in this field.

2. There should be coöperation among radial velocity observers to the ends that duplication may be limited to necessary and desirable degrees, that the resources of the various observatories may be applied chiefly to those problems for which they are the more uniquely adapted, and that the accumulation of accurate and homogeneous results may be expedited. It is not proposed that the individuality of investigators, as to selection of problems and methods, be interfered with, even by suggestion, but only that the evident advantages of coöperation be available to all qualified observers who seek them.

3. A Committee of the American Astronomical Society, embracing the leading radial velocity observers, or their representatives, in 1910–11 gave careful consideration to the proposal that radial velocity determinations should be extended to several thousand stars in the Harvard Revised Photometry fainter than those previously observed for this purpose, on the basis of coöperative division of effort. All of the members but one were of the opinion that, however strongly they might desire to engage in the suggested coöperative plan, their telescopic resources were too weak to give promise of coping successfully with many additional stars. In the meantime a 60-inch and a 73-inch reflecting telescope have engaged heavily in this work, and other great reflectors are under construction or nearing completion. We are of the opinion that coöperation is now practicable and desirable. We recommend that those institutions engaged extensively in measuring the radial velocities of stars should endeavor so to coördinate their programs that the work of determining the radial velocities of the stars in Boss's Preliminary General Catalogue shall be divided amongst them on the basis of their latitudes, the light gathering powers of their telescopes, etc.

4. The present situation in the Southern Hemisphere closely resembles that in the Northern Hemisphere in the year 1910. The intensive use of very large reflecting telescopes in the Northern Hemisphere will soon leave radial velocity determinations in the southern sky far behind, and

thus create a lack of symmetry and balance in the results for the two hemispheres which will impose regrettable limitations upon their statistical power. This will be unfortunate. There is immediate and serious need for increased observing resources in the Southern Hemisphere, based upon the use of reflecting telescopes 60 inches and greater in diameter. Organization for this service should be encouraged. It is mutually understood that the Victoria, Lick, and Mt. Wilson observatories are prepared to form their radial velocity programs in this manner.

5. Observers equipped with medium-sized telescopes should not hold back from radial velocity work because telescopes of much greater power are engaged extensively in this field. They should not be misled into assuming that the relative exposure times vary inversely as the *areas* of the objectives of the telescopes; under average atmospheric conditions the exposures are more nearly as the inverse *diameters* of the objectives. The instruments of medium power may find much useful work to do in observing the brighter stars of Classes O, B and A, whose spectra contain few and broadened lines, with reduced dispersion; and many such stars are waiting observation.

6. The investigation of spectroscopic binaries is a field in which more workers may find fruitful employment. This is especially true of the southern sky. There is need for the application of reflecting telescopes to further study of spectroscopic binary stars in whose spectra the K line of calcium varies but slightly in wave-length. There are, of course, many other minor problems to which radial velocity methods could be applied with promise or certainty of valuable results.

7. Consideration should be given to questions of wave-lengths and methods, to insure that results obtained at all observatories be homogeneous, comparable and reducible to one system. This principle should apply not only to bright stars of a given spectral class, but to bright and faint stars of the same class as observed with spectrographs of widely different dispersions, and as far as possible to stars of all the different spectral classes.

8. There exist at various observatories a great number of spectrograms obtained primarily for radial velocity determinations. We recommend that these spectrograms be utilized for estimates of the absolute magnitudes of the corresponding stars, and therefrom the spectroscopic parallaxes of these stars.

W. W. CAMPBELL, *Chairman*, WALTER S. ADAMS, J. S. PLASKETT.

REPORT OF THE COMMITTEE ON REFORM OF THE CALENDAR

I

At this very unsettled time in the world's history all sorts of reformers, wise and otherwise, are springing up advocating changes in all sorts of

things. It is advisable that scientific organizations of standing be conservative in their advocacy of far-reaching reforms touching the every-day life of the people, *at the present moment*, even though such reforms seem in themselves desirable.

II

The calendar can undoubtedly be revised or reformed to advantage. The following suggestions for such revision or reform, favored by your Committee, are presented for your consideration *without recommendation* as to whether action to accomplish them should be advocated for the present year:

1. The year of twelve months to be divided into four quarters of three months each, and the arrangements for the first quarter to repeat themselves precisely in each of the other quarters. The first month of each quarter to have thirty days; the second month thirty days; and the third month thirty-one days. This accounts for ninety-one days in each quarter, or 364 days in all.

The remaining day in ordinary years to be "New Year's Day." It is to be given no other descriptive title. It is not to belong to any week nor to any month. It begins the year. January 1 to be the day following New Year's Day.

The 366th day of a leap year to be likewise an extra day, bearing an appropriate name (perhaps "Leap Day"), but no week-day name, nor should it be a part of any month. It may be (*a*) the day following December 31, hence the day preceding New Year's Day, or (*b*) the day following June 31, or (*c*) the day following Christmas Day (at the present time the day following Christmas Day is a holiday in some countries). It is assumed to be a holiday—it comes only one year in four.

January 1 to fall on Monday. From this it results that the first month in each quarter (January, April, July, October) begins on Monday in every year; that the second month in each quarter (February, May, August, November) begins always on Wednesday; and that the third month in each quarter (March, June, September, December) begins always on Friday. Further results of this are that (*a*) the first day of the month never falls on Sunday, (*b*) the fifteenth day of the month never falls on Sunday, (*c*) the thirtieth day of the month never falls on Sunday, (*d*) the last day of each quarter (the 31st of March, June, September, December) always falls on Sunday, (*e*) the thirty-day months always have four Sundays each, (*f*) the thirty-one-day months always have five Sundays each, (*g*) the number of week-days in the month is twenty-six for every month, (*h*) holidays *always* fall on the same day of the week. This proposed calendar is outlined in the following table, which will serve for *all years alike*. Leap day is assumed to follow December 31.

New Year's Day																				
First Quarter		January					February						March							
Second Quarter		April					May						June							
Third Quarter		July					August						September							
Fourth Quarter		October					November						December							
Leap Day																				
Monday	1	8	15	22	29		6	13	20	27			4	11	18	25				
Tuesday	2	9	16	23	30		7	14	21	28			5	12	19	26				
Wednesday	3	10	17	24		1	8	15	22	29			6	13	20	27				
Thursday	4	11	18	25		2	9	16	23	30			7	14	21	28				
Friday	5	12	19	26		3	10	17	24		1		8	15	22	29				
Saturday	6	13	20	27		4	11	18	25		2		9	16	23	30				
Sunday	7	14	21	28		5	12	19	26		3		10	17	24	31				

This would be a perpetual calendar, and need be printed but once.

If this calendar should be adopted it should go into effect at the beginning of a year which, according to the present calendar, would begin with Monday in order to avoid such confusion as might arise from the adding or dropping certain days of the beginning week. According to the present calendar the next year that begins on Monday is 1923, and the next following year to begin on Monday is 1934.

It should be pointed out that according to the proposed calendar the Sun will come to the Vernal Equinox about March 19 or 20 instead of March 21 or 22 as under the present calendar.

2. The day to be divided into twenty-four hours numbered consecutively from one to twenty-four, the noon hour to be twelve.

3. The astronomical day to begin at midnight.

R. T. CRAWFORD, *Chairman.*

Signed after expressing very strong preference for "Leap Day" following December 31, in order to break the continuity of the weeks only *once* per annum. W. W. CAMPBELL.

Signed, *excepting* II 2, relating to the 24-hour day, because it would necessitate an impossible change of dials and mechanisms in all existing timepieces. HAROLD JACOBY.

REPORT OF THE COMMITTEE ON RESEARCH SURVEYS

One of the objects of the National Research Council, as expressed in the Executive Order issued by President Wilson on May 11, 1918, is "to survey the larger possibilities of science, to formulate comprehensive projects of research, and to develop effective means of utilizing the scientific and technical resources of the country for dealing with these projects."

The Research Council has already undertaken the preparation of research surveys, pointing out important possibilities in both scientific and industrial research. The chief objects of these surveys are:

1. To aid the specialist in recognizing the wider bearing of his in-

vestigations and the importance of organizing them in such a way as to render his efforts most productive.

2. To indicate the possibility of applying in other branches of science the special instruments, methods, or results developed in particular fields.

3. To formulate large and important research projects, calling for special financial support or for the coöperation of investigators in the same field or of those dealing with other branches of science.

4. To encourage more active investigation in the borderlands between the various branches of science.

The first of the research surveys relating to astronomy was written in 1917 by Dr. Henry Norris Russell, but withheld from publication because of the war. This has recently been revised, and will shortly appear among the Bulletins of the National Research Council under the title "Some Problems of Sidereal Astronomy."[1] The recent report of Dr. C. G. Abbot, entitled "The Larger Opportunities for Research on the Relations of Solar and Terrestrial Radiation,"[2] is also well adapted for publication as a Research Council Bulletin, and your committee recommends that it be brought out in this series in the near future.

We also recommend that Professor E. W. Brown be requested to prepare for publication a survey of the gravitational problems of the solar system. Later Bulletins should deal with cosmogony, the atom and electron in sidereal astronomy, the future possibilities of research in stellar spectroscopy, stellar distances, and other subjects. A synopsis of the adaptation of physical apparatus to astronomical problems would also be a valuable contribution, especially if it indicated some of the astronomical possibilities of recently developed physical methods.

Because of the importance of obtaining the personal views of leading investigators on the opportunities for research in their respective fields, it is not proposed that any single branch of astronomy be exclusively dealt with by a single writer. The suggestive value of these reports is likely to be greater if they are prepared by individuals rather than by committees, as in the latter case the average view, involving the consensus of opinion, is substituted for the special view of men who have approached the subject from different directions.

Your committee requests that it be continued and authorized to arrange for the preparation and publication of research surveys in astronomy, in conformity with any general plan that may be adopted by the National and International Research Councils.

GEORGE E. HALE, *Chairman*, F. R. MOULTON, HARLOW SHAPLEY.

[1] Reprint and Circular Series of the National Research Council, Number 5.

[2] Reprint and Circular Series of the National Research Council, Number 7.